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Welfare Analysis of Fiscal Policies in a Fixed Price Overlapping Generations Model

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Abstract

The purpose of this paper is to study the effects of fiscal policies such as a wasteful public spending and both inter-generational and intra-generational income transfers in a Keynesian under-employment economy. Using a non-Walrasian fixed price overlapping generations model with different groups of households concerning the consumption propensity, we present detailed analyses of the welfare effects of the above policies under both balanced and loan budgets. Our analyses can be regarded as a general equilibrium extension of Ono(2011) which reconsidered the effectiveness of such policies in the simple textbook multiplier model.

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1. Introduction

The textbook Keynesian multiplier analysis states that a fiscal expansion policy can raise GDP and therefore is effective in an under-employment economy, even if it is wasteful in the sense that it contributes to neither utilities of households nor productivities of firms. However, since an increase of GDP does not necessarily mean improving utilities of households, the correctness of the above statement needs to be checked carefully. Recently, Ono(2011) reconsidered the effectiveness of fiscal policies such as a wasteful public spending and an income transfer under the Keynesian multiplier framework, and showed the followings.

(Ⅰ) An increase in a wasteful public spending under a balanced budget raises GDP, but it is not effective because the household's consumption does not change. Namely, it is equivalent to an income transfer between households with the same propensity to consume.

(Ⅱ) An increase in a wasteful public spending under a loan budget raises both GDP and the household's consumption in the presence of fiscal illusion. However, in the absence of fiscal illusion the policy effect of it is equivalent to that under a balanced budget.

(Ⅲ) An income transfer from households with higher propensity to consume to those with lower one reduces GDP. Furthermore, the balanced-budget multiplier can be less than 1 if the government increases a public spending by imposing tax on households with higher propensity to consume.

Ono's analysis is based on the simple textbook multiplier model and therefore it is very brief. Since the non-Walrasian fixed price model, which was pioneered by Clower(1967) and Barro-Grossman(1971, 1976) and was developed by Malinvaud(1977), Benassy(1986, 2002) and so on, provides a proper general equilibrium basis of the multiplier analysis, we can expect that the welfare effect of Keynesian fiscal policies can be examined more precisely by using it. In fact, Benassy(1986, ch3) has already discussed such a question and has shown (Ⅰ) and the first half of (Ⅱ) of Ono's conclusion above. In his analysis, however, the effect of income transfers between different types of households is not considered. Furthermore, since the discussions of both Ono and Benassy are based on a static framework, it is not necessarily clear that what effect such fiscal policies have on the welfare of the next generation. So, in this paper we investigate these issues in detail by using a non-Walrasian overlapping generations model with two different groups of households concerning the propensity to consume[2].

We first consider "Case 1" where at period t the government imposes a lump-sum tax on each group in generation t (young generation at period t), and conducts the following three kinds of policies under a balanced budget: (a) a wasteful public spending, (b) an inter-generational income transfer from generation t to generation $t-1$ (namely, from the young to the old), (c) an intra-generational income transfer between different groups of households in generation t, and demonstrate the followings.

(i) A spending policy (policy (a)) raises GDP. The (balanced-budget) multiplier is 1, but it becomes less than 1 when the government imposes a heavier tax on the group with higher propensity to consume. The consumption of each group in each generation does not change (namely, a policy (A) is neutral from the viewpoint of welfare). Furthermore, the same result holds even when the government implements a spending by borrowing from generation t and repays at the next period by imposing a lump-sum tax to

[2] To my knowledge, Rankin(1986) is the first to present a non-Walrasian OG model. He examined in the model the effects of a permanent increase in public debt stock on capital accumulation and the welfare of future generations in the new steady state.

generation t again (namely, the Ricardian neutrality holds true).

(ⅱ) An inter-generational income transfer from the young to the old (policy (b)) raises GDP by the same size as a policy (a). The consumption of each group in generation t does not change, but that in generation $t-1$ increases, which means that it is a Pareto-improving policy.

(ⅲ) Concerning an intra-generational income transfer (policy (c)), if it is implemented from households with lower (resp. higher) propensity to consume to that with higher (resp. lower) one, then GDP rises (resp. falls). The consumption of the group who receives (resp. does not receive) a transfer necessarily increases (resp. decreases). The consumption of each group in generation $t-1$ remains unchanged.

Among these results, the result (ⅰ) has already been shown by Ono and Benassy, but the results (ⅱ) and (ⅲ) have not yet and they can be interpreted as an extended version of Ono's result (Ⅲ).

We next investigate "Case 2" where the government conducts each policy at period t by issuing public debt (namely, under a loan budget) and repays it at period $t+1$ by imposing a lump-sum tax on each group in generation $t+1$, and show the followings.

(ⅳ) At period t both GDP and the consumption of each group in each generation in Case 2 become larger than in Case 1 in all three policy cases. (This implies that a public spending under a loan budget can raise the consumption at period t even in the absence of fiscal illusion.)

(ⅴ) At period $t+1$ GDP becomes larger in Case 2 than in Case 1 in all three policy cases. The consumption of each group in generation $t+1$ does not change between two cases, despite of the fact that the government shifts a tax burden to each group in generation $t+1$ in Case 2 while it does not in Case 1.

Accordingly, in all three policy cases the government can improve the welfare in the sense of Pareto by shifting a tax burden to the next generation. This is because such a shift is essentially equivalent to implementing an income transfer from the young to the old at period $t+1$ and such an inter-generational transfer can improve the consumption of the young without harming that of the old, as shown in the result (ⅱ) of Case 1. This result means that under a non-Walrasian fixed price model there is no burden of public debt, which is investigated in more general setups by Tanaka(2008) and Ogawa and Ono(2010)[3].

The organization of the rest of this paper is as follows. In section 2, before proceeding to the policy analysis, we present the basic structure of the "benchmark case" where the government activity is omitted. In section 3 and section 4 we examine "Case 1" and "Case 2" mentioned above, and prove the results (ⅰ)～(ⅴ) rigorously. Finally, in section 5 we conclude the paper.

2. The Benchmark Case

In this section we present the basic structure of the benchmark case without the government sector before proceeding to the policy analysis. We consider a discrete-time closed-economy overlapping-generations model where each generation lives for two periods (young and old). Each generation consists of two different groups (the group A and the group B) of households who are identical except for the form of utility function (in other words, the propensity to consume). Households of the group A (resp. the

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[3] Especially, Ogawa and Ono (2010) shows the result under a fairly general framework of Rankin(1986)'s type where the household has a money in the utility (MIU) function and the firm makes an investment decision.

group B) exist in a continuum at the interval $[0, h]$ (resp. the interval $[h,1]$) where 0 $< h <$ 1, so the total population size of each generation is 1 and constant over time. It is assumed that there is no capital good in this economy and the only way of saving is to hold a fiat money, so young households spend a part of their income to purchasing money for the purpose of saving and then they sell it at their old period.

Since the purpose of this paper is to investigate the welfare effects of fiscal policies in a Keynesian under-employment economy, we adopt a non-Walrasian fixed price model where both price and wage are exogenously fixed and accordingly the "equilibrium" of the (output) market is achieved by quantity adjustment. We especially focus on the regime of so called "Keynesian unemployment", where both the output and labor markets face excess supplies and therefore young households (resp. firms) supply the labor (resp. the output) which quantity is equal to the firm's labor demand (resp. the household's aggregate output demand).

In what follows we formulate the behavior of each agent in detail, and then derive the non-Walrasian equilibrium.

Households of generation t

At the beginning of period t each household of generation t is endowed with L units of labor. It is assumed, however, that because the fixed real wage is high and accordingly the firm's labor demand (L_t^d) is smaller than L at the wage level, each household is forced to supply L_t^d units of labor (in other words, each household faces "demand constraint" in the labor market).

The utility function of the group $j(=A, B)$'s household is denoted by $U_{t,j}$ =

$U_j(C^y_{t,j}, C^o_{t+1,j})$, where $C^y_{t,j}$ (resp. $C^o_{t+1,j}$) is the young (resp. old) period consumption of the group j at period t (resp. $t+1$). For simplicity of analysis the disutility of labor supply is not considered[4]. We assume that $U_{t,j}$ is homothetic and satisfies the usual list of conditions. Each household of the group j maximizes $U_{t,j}$ under the following budget constraint:

(1) $$P_t C^y_{t,j} + M^d_{t,j} = W_t L^d_t + \Pi_t, \quad P_{t+1} C^o_{t+1,j} = M^d_{t,j},$$

where $M^d_{t,j}$ is the nominal money demand, P_t is the nominal price, W_t is the nominal wage, and Π_t is the nominal profit. For simplicity, we assume throughout the paper that both nominal price and wage are fixed at a constant level.

(2) $$P_t = P, \quad W_t = W$$

So the budget constraint (1) can be rewritten as

$$C^y_{t,j} + m^d_{t,j} = wL^d_t + \pi_t, \quad C^o_{t+1,j} = m^d_{t,j},$$

where $m^d_{t,j} \equiv M^d_{t,j}/P$, $w \equiv W/P$ and $\pi_t \equiv \Pi_t/P$. From the assumption of the homothetic utility function the optimal consumption and saving plans of the group $j(=A,B)$'s household can be derived as

(3) $$C^y_{t,j} = \alpha_j [wL^d_t + \pi_t] \quad m^d_{t,j} = (1-\alpha_j)[wL^d_t + \pi_t],$$

where we assume $0 < \alpha_j < 1$.

Households of generation $t-1$

At the beginning of period t households of the group $j(=A,B)$ in generation $t-1$

[4] Since we focus on an under-employment situation where households works less than they want to do, such an assumption is not problematic.

have $m^d_{t-1,j}$ units of money which was stored in their young period, and they spend all of them to their old period consumptions. So their behaviors at period t can be expressed by

(4) $$C^o_{t,j} = m^d_{t-1,j} .$$

Note that the following relationship holds:

(5) $$hm^d_{t-1,A} + (1-h)m^d_{t-1,B} \equiv m ,$$

where m denotes the exogenous real money stock, which is assumed to be fixed at a constant level.

Firms

There are many identical firms and the number of them is normalized to 1. Each firm has the usual concave production function $Y_t = F(L_t)$, where Y_t is the output and L_t is the labor input. Since the real profit of each firm is defined as $\pi_t \equiv F(L_t) - wL_t$, the firm's optimal labor demand and output supply would be

(6) $$F'(L_t) = w \;\; \rightarrow \;\; L_t = F'^{-1}(w) \;\; \text{and} \;\; Y_t = F[F'^{-1}(w)],$$

if the optimal output supply Y_t was smaller than the aggregate output demand Y^d_t :

(7) $$Y^d_t = [hC^y_{t,A} + (1-h)C^y_{t,B}] + [hC^o_{t,A} + (1-h)C^o_{t,B}].$$

We assume, however, that the firm faces output demand constraint (namely, $Y^d_t < Y_t$) due to a high fixed real wage and therefore each firm is forced to supply Y^d_t units of output. So, the actual labor demand and real profit are

(8) $$Y^d_t = F(L^d_t) \;\; \rightarrow \;\; L^d_t = L^d_t(Y^d_t), \;\; \pi_t = F(L^d_t) - wL^d_t .$$

Non-Walrasian market equilibrium

Substituting the third equation in (8) into the first equation of (3), the consumption of the group $j(=A,B)$ of generation t can be rewritten as

(9) $$C_{t,j}^{y} = \alpha_j Y_t^d .$$

From (4) and (5) the aggregate consumption demand of generation $t-1$ is

(10) $$hC_{t,A}^{o} + (1-h)C_{t,B}^{o} = m$$

Thus, by substituting (9) and (10) into (8), we can derive the "equilibrium" output in the benchmark case as

(11) $$Y_t^{bench} = m/\Delta . \qquad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B),\ 0 < \Delta < 1)$$

Here, Δ can be interpreted as the saving rate at a macro-level. From (11) we can confirm the followings.

・ An increase in the exogenous real money stock m raises the equilibrium output, and the multiplier (dY_t^{bench}/dm) is $1/\Delta\,(>1)$

・ An increase in the saving rate Δ reduces the equilibrium output (namely, "the paradox of thrift" holds).

The young and old period equilibrium consumptions of the group $j(=A,B)$ of generation t are

(12) $$(C_{t,j}^{y})^{bench} = \alpha_j m/\Delta , \quad (C_{t+1,j}^{o})^{bench} \ \ (=(m_{t,j}^{d})^{bench}) \ = (1-\alpha_j)m/\Delta .$$

The old period consumption of the group $j(=A,B)$ in generation $t-1$, on the other hand, is given by

(13) $$(C_{t,j}^{o})^{bench} = (1-\alpha_j)m/\Delta ,$$

because the same equilibrium state is reproduced at every period in the benchmark case[5] and therefore $(C^o_{t,j})^{bench} = (C^o_{t+1,j})^{bench}$ holds true.

(Remark)

The regime of "Keynesian unemployment" discussed above holds under the conditions of $L^d_t < L$ in the labor market and $Y^d_t < Y_t = F[F'^{-1}(w)]$ in the output market. Since $L^d_t < L$ can be rewritten as $F(L^d_t) = Y^d_t < F(L)$ and Y^d_t is equal to m/Δ in equilibrium, the range of exogenous price (P) and wage (W) under which such a scheme is realized is $\frac{M}{P\Delta} < F(L)$ and $\frac{M}{P\Delta} < F\left[F'^{-1}\left(\frac{W}{P}\right)\right]$.

2. Case 1

In this section we introduce the government sector into the benchmark case of the previous section and investigate the welfare effects of various fiscal policies. More concretely, in this section we consider "Case 1" where at period t the government imposes a lump-sum tax on each group in generation t and conducts the following three kinds of policies under a balanced budget: (a) a wasteful public spending, (b) an inter-generational income transfer from generation t to generation $t-1$ (namely, from the young to the old), (c) an intra-generational income transfer between different groups in generation t. The main conclusions of this section can be summarized as follows.

[5] This is because the real money stock (the only stock variable in our model) is constant over time.

(ⅰ) A spending policy (policy (a)) raises GDP, but the consumption of each group in each generation does not change, which means that it is neutral from the viewpoint of welfare.

(ⅱ) An inter-generational income transfer (policy (b)) raises GDP by the same size as a policy (a). The consumption of each group in generation t does not change, but that in generation $t-1$ rises, which means that it is a Pareto-improving policy.

(ⅲ) Concerning an intra-generational income transfer (policy (c)), if it is implemented from the group with lower (resp. higher) propensity to consume to that with higher (resp. lower) one, then GDP rises (resp. falls). The consumption of the group who receives (resp. does not receive) a transfer necessarily increases (resp. decreases). The consumption of each group in generation $t-1$ remains unchanged.

The government

At period t the government imposes a lump-sum tax τ in real terms on each group in generation t, so the total revenue is $h\tau + (1-h)\tau = \tau$. This revenue is used for (a) a wasteful public spending, or (b) an inter-generational transfer, or (c) an intra-generational transfer.

Households of generation t

In Case 1 the problem of the group A's household in generation t is formulated as

$$\max_{C^y_{t,A}, C^o_{t+1,A}} U_{t,A} \qquad \text{s.t.} \quad C^y_{t,A} + m^d_{t,A} = wL^d_t + \pi_t - \tau + \delta_1 \frac{\tau}{h}, \quad C^o_{t+1,A} = m^d_{t,A},$$

where δ_1 is a dummy variable which is equal to 1 when an intra-generational transfer (policy (c)) is implemented and 0 otherwise. (Notice that an intra-generational transfer is implemented from the group B to A.) The optimal consumption and saving plans are

(14) $$C^{y}_{t,A} = \alpha_A [wL^d_t + \pi_t - \tau + \delta_1 \frac{\tau}{h}], \quad m^d_{t,A} = (1-\alpha_A)[wL^d_t + \pi_t - \tau + \delta_1 \frac{\tau}{h}].$$

The problem of the group B, on the other hand, is given by

$$\max_{C^y_{t,B}, C^o_{t+1,B}} U_{t,B} \qquad \text{s.t.} \quad C^y_{t,B} + m^d_{t,B} = wL^d_t + \pi_t - \tau, \quad C^o_{t+1,B} = m^d_{t,B},$$

so the optimal consumption and saving plans are

(15) $$C^y_{t,B} = \alpha_B [wL^d_t + \pi_t - \tau], \quad m^d_{t,B} = (1-\alpha_B)[wL^d_t + \pi_t - \tau].$$

Households of generation $t-1$

Each household in generation $t-1$ receives a transfer which size is τ in real terms if the government implements an inter-generational transfer (a policy (b)). She spends all her incomes (the sum of an initial money holding and a transfer) to the old period consumption, so the behavior of her can be expressed by

(16) $$C^o_{t,j} = m^d_{t-1,j} + \delta_2 \tau, \qquad (j = A, B)$$

where δ_2 is a dummy variable which is equal to 1 when an inter-generational transfer is implemented and 0 otherwise.

Firms

The behavior of the firm is the same as the benchmark case, so the actual labor demand and real profit are

(17) $$Y^d_t = F(L^d_t) \quad \rightarrow \quad L^d_t = L^d_t(Y^d_t), \quad \pi_t = F(L^d_t) - wL^d_t.$$

Non-Walrasian market equilibrium

In what follows we derive the non-Walrasian equilibrium in each case of three policies in turn.

(a) The case of a wasteful public spending

In this case $\delta_1 = \delta_2 = 0$ holds and the size of a public spending is τ , so the aggregate demand for output is given by

(18) $$Y_t^d = [hC_{t,A}^y + (1-h)C_{t,B}^y] + [hC_{t,A}^o + (1-h)C_{t,B}^o] + \tau \, .$$

Here, notice that a public spending τ is newly added in the component of aggregate output demand. From (14)～(17), the consumption of each group $j = A, B$ in each generation can be rewritten as

(19) $$C_{t,j}^y = \alpha_j [Y_t^d - \tau], \quad C_{t,j}^o = m_{t-1,j}^d . \quad \text{(where } hm_{t-1,A}^d + (1-h)m_{t-1,B}^d \equiv m \text{)} \, .$$

So the equilibrium output in Case 1 (a) is

(20) $$Y_t^{1,a} = \frac{m}{\Delta} + \tau \, . \quad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

From (11) and (20), the equilibrium output in Case 1 (a) is larger just by the size of a public spending than in the benchmark case, which means that the balanced budget multiplier is equal to 1. The reason why $Y_t^{1,a} > Y_t^{bench}$ holds is as follows. In Case 1 (a) the consumption demand of each group in generation t lowers due to an imposition of tax, but its extent is smaller than the size of tax (or public spending) itself because of the inter-temporal consumption smoothing behavior (see (19)). As a result, the aggregate demand for output in Case 1 (a) is more stimulated than the benchmark case and the equilibrium output becomes higher.

However, this result does not necessarily means that a wasteful public spending policy is welfare-improving, because the welfare of each household depends not on GDP but on the equilibrium consumption level. Substituting (20) into (19), we have

(21) (generation t) $(C^y_{t,j})^{1,a} = \alpha_j m/\Delta$, $(C^o_{t+1,j})^{1,a} = (1-\alpha_j)m/\Delta$,

(generation $t-1$) $(C^o_{t,j})^{1,a} = (1-\alpha_j)m/\Delta$. $(j = A, B)$

From (12), (13) and (21) the consumption level of each group in each generation is the same between Case 1 (a) and the benchmark case. It is especially notable that the consumptions of generation t are the same despite the fact that generation t bears a tax in Case 1 (a) while it does not in the benchmark case. The reason why it holds is that in Case 1 (a) a tax burden is exactly offset by an increase of income caused by a rise in the equilibrium output.

After period $t+1$ the equilibrium states are the same between two cases, because the real money stock m (the only stock variable in our model) is constant and the government plays no role after the period. We can therefore conclude that a wasteful public spending (policy (a)) under a balanced budget is neutral from the viewpoint of welfare. This result is plausible in a sense, because the increased output caused by a public spending is used for a wasteful purpose and the amount of output utilized for consumption remains unchanged between two cases.

(Remark 1)

We can easily show that the same result holds even if we consider the case where the government implements a public spending by borrowing from each group in generation t and repays at the next period by imposing a lump-sum tax to generation t again. That is to say, the timing of taxation (namely, whether it is imposed at the young or old period) has no influence on the consequence as far as the tax burden is attributed to an identical household, which means that the Ricardian Neutrality holds in our model.

(Remark 2)

We can obtain the different result from Case 1 (a) in the case where the government imposes the different size of a lump-sum tax on each group in generation t. Consider the situation where the government imposes τ_A (resp. τ_B) units of tax on the group A (resp. B) in generation t while the total amount of tax revenue is fixed at the same level as Case 1 (a) (namely, $h\tau_A + (1-h)\tau_B = \tau$). In this situation the equilibrium output can be calculated as

$$Y_t = \frac{m}{\Delta} + \frac{\tau - [h\alpha_A\tau_A + (1-h)\alpha_B\tau_B]}{\Delta}. \quad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

Notice that this is reduced to (20) if $\tau_A = \tau_B = \tau$ holds. The balanced budget multiplier when the additional spending is financed from the group A only is

$$\left.\frac{dY_t}{d\tau}\right|_{d\tau_B=0} = \frac{1-\alpha_A}{\Delta}.$$

This multiplier becomes less than 1 if the propensity to consume of the group A is larger than the group B (namely, $\alpha_A > \alpha_B$), as was pointed out by Ono(2011). This result shows that the balanced budget multiplier becomes lower when the additional spending is financed from the group with higher propensity to consume.

The equilibrium consumption of each group in generation t is

$$C_{t,A}^y = \alpha_A\left[\frac{m}{\Delta} + \frac{(1-h)(1-\alpha_B)(\tau_B - \tau_A)}{\Delta}\right], \quad C_{t+1,A}^o = \frac{1-\alpha_A}{\alpha_A} C_{t,A}^y$$

$$C_{t,B}^y = \alpha_B\left[\frac{m}{\Delta} + \frac{h(1-\alpha_A)(\tau_A - \tau_B)}{\Delta}\right], \quad C_{t+1,B}^o = \frac{1-\alpha_B}{\alpha_B} C_{t,B}^y$$

From (21), both the young and old period consumptions of the group A (resp. B) decrease (resp. increases) by a public spending policy if the tax burden of the group A

is larger than the group B (namely, $\tau_A > \tau_B$). Accordingly, the result in Case 1 (a) that a wasteful public spending is neutral from the viewpoint of welfare does not hold when the different size of tax is imposed on each group in generation t.

(b) The case of an inter-generational transfer

In this case $\delta_1 = 0$ and $\delta_2 = 1$ hold and the size of a public spending is zero, so the aggregate demand for output is

(22) $$Y_t^d = [hC_{t,A}^y + (1-h)C_{t,B}^y] + [hC_{t,A}^o + (1-h)C_{t,B}^o].$$

From (14)～(17), the consumption of each group $j = A, B$ in each generation is

(23) $$C_{t,j}^y = \alpha_j [Y_t^d - \tau], \quad C_{t,j}^o = m_{t-1,j}^d + \tau. \quad (\text{where } hm_{t-1,A}^d + (1-h)m_{t-1,B}^d \equiv m)$$

Accordingly, the equilibrium output in Case 1 (b) is

(24) $$Y_t^{1,b} = \frac{m}{\Delta} + \tau. \quad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

From (20) and (24), the equilibrium output in Case 1 (b) is equal to that in Case 1 (a). This is because in the case of an inter-generational transfer all of the transferred incomes to old generation are spent to consumption expenditure and therefore the aggregate demand level of this case is equivalent to Case 1 (a) where all of the tax revenue is spent for government purchase.

Concerning the equilibrium consumption of each group $j = A, B$ in each generation, we have

(25) (generation t) $$(C_{t,j}^y)^{1,b} = \alpha_j m/\Delta, \quad (C_{t+1,j}^o)^{1,b} = (1-\alpha_j)m/\Delta$$

(generation $t-1$) $$(C_{t,j}^o)^{1,b} = (1-\alpha_j)\frac{m}{\Delta} + \tau.$$

From (21) and (25), both the young and old period consumptions of each group in generation t are the same between Case 1 (b) and Case 1 (a), but the consumption of each group in generation $t-1$ becomes higher in Case 1 (b) due to the transfer. Since after period $t+1$ the equilibrium states are identical between two cases, we can conclude that Case 1 (b) is Pareto-superior to Case 1 (a). This result stems from the fact that in Case 1 (b) the tax revenue from generation t is utilized to raise the consumption of generation $t-1$ while in Case 1 (a) it is used for a wasteful public spending.

(Remark)

We can easily confirm that as far as the total amount of a transfer is fixed the equilibrium output (24) does not change even when the size of a transfer to each group in generation $t-1$ is different. (For example, each household of the group A receives τ / h units of transfers while each household of the group B does not receive any transfers). This is because the consumption propensity of both groups in generation $t-1$ is 1 and therefore the total size of consumption demand of generation $t-1$ does not change.

(c) The case of an intra-generational transfer

In this case $\delta_1 = 1$ and $\delta_2 = 0$ hold and the size of a public spending is zero, so the aggregate demand is

(26) $$Y_t^d = [hC_{t,A}^y + (1-h)C_{t,B}^y] + [hC_{t,A}^o + (1-h)C_{t,B}^o].$$

From (14)～(17), the consumption of each group $j = A, B$ in each generation is

(27) $$C_{t,A}^y = \alpha_A [Y_t^d - \tau + \frac{\tau}{h}], \quad C_{t,B}^y = \alpha_B [Y_t^d - \tau], \quad C_{t,j}^o = m_{t-1,j}^d .$$

So, the equilibrium output in Case 1 (c) is

(28) $$Y_t^{1,c} = \frac{m}{\Delta} + \frac{(1-h)(\alpha_A - \alpha_B)\tau}{\Delta}. \qquad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

From (11) and (28), in the case of $\alpha_A > \alpha_B$ (namely, the group A's propensity to consume is larger than the group B's), the equilibrium output in Case 1 (c) becomes higher than in the benchmark case. This is because an income transfer from the group with lower propensity to consume to the group with higher one stimulates the aggregate consumption demand. However, even in such a case the equilibrium output in Case 1 (c) is lower than Case 1 (a) or Case 1 (b), because in Case 1 (a) or Case 1 (b) all of the tax revenue is spent for government purchase or old period consumption while in Case 1 (c) a part of the transferred tax revenue to the group A in generation t is saved.

Concerning the equilibrium consumptions of each group in each generation, we have

(29) (generation $t-1$) $$(C_{t,j}^{o})^{1,c} = (1-\alpha_j)\frac{m}{\Delta} \qquad (j = A, B)$$

(generation t)

(group A) $$(C_{t,A}^{y})^{1,c} = \alpha_A\left[\frac{m}{\Delta} + \frac{(1-h)(1-\alpha_B)\tau}{h\Delta}\right], \quad (C_{t+1,A}^{o})^{1,c} = \frac{1-\alpha_A}{\alpha_A}(C_{t,A}^{y})^{1,c}$$

(group B) $$(C_{t,B}^{y})^{1,c} = \alpha_B\left[\frac{m}{\Delta} - \frac{(1-\alpha_A)\tau}{\Delta}\right], \quad (C_{t+1,B}^{o})^{1,c} = \frac{1-\alpha_B}{\alpha_B}(C_{t,B}^{y})^{1,c}.$$

From (12), (13) and (29), the consumption of each group in generation $t-1$ is equal between Case 1 (c) and the benchmark case, and the young and old consumptions of the group A who receives a transfer (resp. the group B who does not receive it) is higher (resp. lower) in Case 1 (c) than in the benchmark case. Note that this result does not depend on magnitude relation of each group's consumption propensity. In the case of $\alpha_A < \alpha_B$ (resp. $\alpha_A > \alpha_B$) the equilibrium output decreases (resp. increases) relative to the benchmark case, but nevertheless the consumptions of the group A who receives a transfer becomes higher (resp. the group B who does not receive it becomes lower) than the benchmark case. In other words, the size of a transfer is necessarily larger than the variation of the equilibrium output (income) caused by a policy (c).

4. Case 2

In this section we consider "Case 2" where the government implements three kinds of fiscal policies (a)～(c) at period t by borrowing from each group in generation t and repays at the next period by imposing a lump-sum tax to each group in generation $t+1$ (the next generation), and re-examine the welfare effect of each policy. The main conclusions of this section can be summarized as follows.

(ⅳ) At period t both GDP and the consumption of each group in generation t become larger in Case 2 than in Case 1 in all three policy cases.

(ⅴ) At period $t+1$ GDP becomes larger in Case 2 than in Case 1 in all three policy cases. The consumption of each group in generation $t+1$ is equivalent between two cases, despite of the fact that a tax burden is shifted to each group of generation $t+1$ in Case 2 while it is not in Case 1. This means that a shift of a tax burden to the next generation is a Pareto-improving policy.

4.1 Equilibrium at period t

The government

The government issues τ units of public debt in real terms (which is equal to the tax revenue in Case 1) and sells them to each group in generation t. Denoting the amount of public debt purchased by the group A and B as $b_{t,A}$ and $b_{t,B}$ respectively, the following holds: $\tau = hb_{t,A} + (1-h)b_{t,B}$. This revenue is utilized for (a) a wasteful public spending, or (b) an inter-generational transfer, or (c) an intra-generational transfer.

Households in generation t

In Case 2 the problem of the group A's household in generation t is

$$\max_{C^y_{t,A}, C^o_{t+1,A}} U_{t,A} \qquad \text{s.t.} \quad C^y_{t,A} + m^d_{t,A} + b_{t,A} = wL^d_t + \pi_t + \delta_1 \frac{\tau}{h}, \quad C^o_{t+1,A} = m^d_{t,A} + b_{t,A}.$$

Note that in Case 2 there are two ways of saving: money and public debt. Since the price of output is assumed to be constant over time (see (2)) and therefore the return rate of holding money is zero, then the return rate of public debt must be also zero by the arbitrage condition. The optimal consumption and saving plans of the group A are

(30) $$C^y_{t,A} = \alpha_A [wL^d_t + \pi_t + \delta_1 \frac{\tau}{h}], \quad m^d_{t,A} + b_{t,A} = (1-\alpha_A)[wL^d_t + \pi_t + \delta_1 \frac{\tau}{h}].$$

The problem of the group B is given by

$$\max_{C^y_{t,B}, C^o_{t+1,B}} U_{t,B} \qquad \text{s.t.} \quad C^y_{t,B} + m^d_{t,B} + b_{t,B} = wL^d_t + \pi_t, \quad C^o_{t+1,B} = m^d_{t,B} + b_{t,B},$$

so the optimal consumption and saving plans are

(31) $$C^y_{t,B} = \alpha_B [wL^d_t + \pi_t], \quad m^d_{t,B} + b^d_{t,B} = (1-\alpha_B)[wL^d_t + \pi_t].$$

Households in generation $t-1$ and Firms

The behaviors of each group $j = A, B$ in generation $t-1$ and firms are the same as Case 1, so we have

(32) $$C^o_{t,j} = m^d_{t-1,j} + \delta_2 \tau,$$

(33) $$Y^d_t = F(L^d_t) \quad \rightarrow \quad L^d_t = L^d_t(Y^d_t), \quad \pi_t = F(L^d_t) - wL^d_t.$$

Non-Walrasian market equilibrium

In what follows we derive the non-Walrasian equilibrium in each case of three policies in turn.

(a) The case of a wasteful public spending

In this case $\delta_1 = \delta_2 = 0$ holds and the size of a public spending is τ , so the aggregate demand for output is given by (18), and from (30)～(33) the consumption of each group $j = A, B$ in each generation is

(34) $$C^y_{t,j} = \alpha_j Y^d_t , \quad C^o_{t,j} = m^d_{t-1,j} . \quad \text{(where } hm^d_{t-1,A} + (1-h)m^d_{t-1,B} \equiv m \text{)}$$

Thus, the equilibrium output in Case 2 (a) can be calculated as

(35) $$Y^{2,a}_t = \frac{m}{\Delta} + \frac{\tau}{\Delta} . \quad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

From (20) and (35), the equilibrium output at period t in Case 2 (a) is larger than in Case 1 (a), which means that shifting a tax burden to the next generation stimulates the output. The reason is that in Case 2 (a) each group in generation t does not bear a tax burden and therefore the consumption demand is larger than in Case 1 (a) (see (19) and (34)).

Concerning the equilibrium consumption of each group $j = A, B$ in each generation, we have

(36) $$\text{(generation } t\text{)} \quad (C^y_{t,j})^{2,a} = \alpha_j \left(\frac{m}{\Delta} + \frac{\tau}{\Delta} \right), \quad (C^o_{t+1,j})^{2,a} = \frac{1-\alpha_j}{\alpha_j} (C^y_{t,j})^{2,a}$$

$$\text{(generation } t-1\text{)} \quad (C^o_{t,j})^{2,a} = (1-\alpha_j) \frac{m}{\Delta} .$$

From (21) and (36) the young and old period consumptions of each group in generation t are higher in Case 2 (a) than in Case 1 (a), while the consumption of each group in generation $t-1$ is the same between two cases.

(b) The case of an inter-generational transfer

In this case $\delta_1 = 0$ and $\delta_2 = 1$ hold and the size of a public spending is zero, so the aggregate demand for output is given by (22), and from (30)～(33) the consumption of each group $j = A, B$ in each generation is

(37) $$C^y_{t,j} = \alpha_j Y^d_t\,, \quad C^o_{t,j} = m^d_{t-1,j} + \tau\,.$$

Hence, the equilibrium output in Case 2 (b) is

(38) $$Y^{2,b}_t = \frac{m}{\Delta} + \frac{\tau}{\Delta}. \quad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

The equilibrium output at period t in Case 2 (b) becomes larger than in Case 1 (b) (see (24) and (38)) by the same reason as in Case 2 (a) discussed above. The equilibrium consumption of each group $j = A, B$ in each generation is

(39) (generation t) $$(C^y_{t,j})^{2,b} = \alpha_j\left(\frac{m}{\Delta} + \frac{\tau}{\Delta}\right), \quad (C^o_{t+1,j})^{2,b} = \frac{1-\alpha_j}{\alpha_j}(C^y_{t,j})^{2,b},$$

(generation $t-1$) $$(C^o_{t,j})^{2,b} = (1-\alpha_j)\frac{m}{\Delta}.$$

From (25) and (39) the consumptions in generation t are higher in Case 2 (b) than in Case 1 (b) while those in generation $t-1$ are the same between two cases, as in Case 2 (a).

(c) The case of an intra-generational transfer

In this case $\delta_1 = 1$ and $\delta_2 = 0$ hold and the size of a public spending is zero, so the aggregate demand for output is given by (26) and the consumption of each group in each generation is

(40) $$C^y_{t,A} = \alpha_A [Y^d_t + \frac{\tau}{h}], \quad C^y_{t,B} = \alpha_B Y^d_t, \quad C^o_{t,j} = m^d_{t-1,j}. \quad (j = A, B)$$

Accordingly, the equilibrium output in Case 2 (c) is

(41) $$Y^{2,c}_t = \frac{m}{\Delta} + \frac{\alpha_A \tau}{\Delta}. \qquad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

The equilibrium output at period t in Case 2 (c) becomes larger than in Case 1 (c) (see (28) and (41)) by the same reason as in Case 2 (a). The equilibrium consumption of each group in each generation is

(42) (generation t)

$$(C^y_{t,A})^{2,c} = \alpha_A \left[\frac{m}{\Delta} + \left(\frac{1-\alpha_B}{h} + \alpha_B \right) \frac{\tau}{\Delta} \right], \quad (C^o_{t+1,A})^{2,c} = \frac{1-\alpha_A}{\alpha_A} (C^y_{t,A})^{2,c}$$

$$(C^y_{t,B})^{2,c} = \alpha_B \left(\frac{m}{\Delta} + \frac{\alpha_A \tau}{\Delta} \right), \quad (C^o_{t+1,B})^{2,c} = \frac{1-\alpha_B}{\alpha_B} (C^y_{t,B})^{2,c},$$

(generation $t-1$) $$(C^o_{t,j})^{2,c} = (1-\alpha_j) \frac{m}{\Delta}. \qquad (j = A, B)$$

The consumptions in generation t are higher in Case 2 (c) than in Case 1 (c) while those in generation $t-1$ are the same between two cases, as in Case 2 (a).

(Remark)

When the government transfers the revenue financed by debt issue not to the group A only but to each group equally, the young period consumption of each group $j = A, B$ is $C^y_{t,j} = \alpha_j [Y^d_t + \tau]$, and the equilibrium output in such a situation is

$$Y_t = \frac{m}{\Delta} + \frac{[h\alpha_A + (1-h)\alpha_B]\tau}{\Delta}. \qquad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

By comparing this with (42), we have

$$\alpha_A \overset{>}{\underset{<}{=}} \alpha_B \quad \Leftrightarrow \quad Y^{2,c}_t \overset{>}{\underset{<}{=}} Y_t.$$

This shows that if the consumption propensity of the group A is larger (resp. smaller) than the group B, the equilibrium output when the government transfers the revenue to the group A only is larger (resp. smaller) than that when it does to each group equally. The equilibrium consumption of each group $j = A, B$ in each generation is

$$\text{(generation } t\text{)} \quad C^y_{t,j} = \alpha_j \left(\frac{m}{\Delta} + \frac{\tau}{\Delta} \right), \quad C^o_{t+1,j} = \frac{1-\alpha_j}{\alpha_j} C^y_{t,j},$$

$$\text{(generation } t-1\text{)} \quad C^o_{t,j} = (1-\alpha_j)\frac{m}{\Delta}.$$

The young and old period consumptions of the group A (resp. the group B) in generation t become lower (resp. higher) than Case 2 (c), while the consumption of each group in generation $t-1$ remains unchanged. Notice that this equilibrium consumption allocation is the same as that in Case 2 (a).

4.2 Equilibrium at period $t+1$

In the previous subsection 4.1 we showed that both the equilibrium output at period t and consumption of each group in generation t become larger in Case 2 than in Case 1 in all three policy cases. However, in Case 2 the welfare of each group in generation $t+1$ may be harmed since a tax burden is shifted to her. In this subsection we examine the effect of such a tax shift on the equilibrium output at period $t+1$ and the welfare of generation $t+1$.

The government

At period $t+1$ the government imposes a lump-sum tax τ on each group in generation $t+1$ (so the total tax revenue is also τ), and repays a debt to each group in generation t.

Households in generation $t+1$

The problem of each group $j = A, B$ in generation $t+1$ is

$$\max_{C^y_{t+1,j},\,C^o_{t+2,j}} \quad U_{t+1,j} = U_j(C^y_{t+1,j}, C^o_{t+2,j})$$

$$\text{s.t.} \quad C^y_{t+1,j} + m^d_{t+1,j} = wL^d_{t+1} + \pi_{t+1} - \tau\,, \quad C^o_{t+2,j} = m^d_{t+1,j}\,.$$

So, the optimal consumption and saving plans are

(43) $$C^y_{t+1,j} = \alpha_j\,[\,wL^d_{t+1} + \pi_{t+1} - \tau\,], \quad m^d_{t+1,j} = (1-\alpha_j)\,[\,wL^d_{t+1} + \pi_{t+1} - \tau\,].$$

Households in generation t

Each group in generation t spends all her incomes (the sum of a money holding and a debt repayment) to the old period consumption. Although the levels of old period consumptions are different among three policy cases (see (36), (39) and (42)), but the aggregate level becomes equal among them:

(44) $$hC^o_{t+1,A} + (1-h)C^o_{t+1,B} = [\,h(1-\alpha_A) + (1-h)(1-\alpha_B)\,]\left(\frac{m}{\Delta} + \frac{\tau}{\Delta}\right).$$

Namely, the aggregate consumption of generation t at period $t+1$ does not depend on which policy is implemented at period t.

Firms

The behavior of the firm is the same as before, so we have

(45) $$Y^d_{t+1} = F(L^d_{t+1}) \quad \rightarrow \quad L^d_{t+1} = L^d_{t+1}(Y^d_{t+1})\,, \quad \pi_{t+1} = F(L^d_{t+1}) - wL^d_{t+1}$$

Non-Walrasian market equilibrium

The aggregate demand for output at period $t+1$ is given by

$$Y_{t+1}^{d} = [hC_{t+1,A}^{y} + (1-h)C_{t+1,B}^{y}] + [hC_{t+1,A}^{o} + (1-h)C_{t+1,B}^{o}].$$

Here, from (43) and (45) the consumption of each group $j = A, B$ in generation t is

$$C_{t+1,j}^{y} = \alpha_j [Y_{t+1}^{d} - \tau],$$

and the aggregate consumption of generation t is given by (44). So the equilibrium output at period $t+1$ in Case 2 can be derived as

$$Y_{t+1}^{2} = \frac{m}{\Delta} + \tau . \qquad (\Delta \equiv h(1-\alpha_A) + (1-h)(1-\alpha_B))$$

This is larger than the equilibrium output at period $t+1$ in Case 1 (which is equal to m/Δ in all three policy cases). The equilibrium young and old period consumptions of each group $j = A, B$ in generation $t+1$ are

$$(C_{t+1,j}^{y})^2 = \alpha_j \frac{m}{\Delta}, \quad (C_{t+2,j}^{o})^2 = (1-\alpha_j)\frac{m}{\Delta}.$$

This result shows that in spite of the fact that a tax burden is shifted to each group in generation $t+1$ in Case 2 while it is not in Case 1, the consumption of each group in generation $t+1$ is equal between two cases. Why does such a result hold? This is because a shift of the tax burden to generation $t+1$ is equivalent to an income transfer from (the young) generation $t+1$ with lower consumption propensity to (the old) generation t with higher one, and that such an inter-generational transfer brings about an increase of the equilibrium output which just offsets the tax burden of generation $t+1$ as is closely discussed in Case 1 (b).

After period $t+2$ the equilibrium states are identical between Case 2 and Case 1, because the real money stock m is constant and the government plays no role after the period. We can therefore conclude that in all three policy cases Case 2 is Pareto-superior to Case 1. That is to say, in a fixed price model the government can improve the welfare of generation t without harming the welfare of future generations by shifting a tax burden to generation $t+1$, which is shown also in Tanaka (2008) and Ogawa and Ono(2010) in more general setups.

5. Final Remarks

We examined the effects of three kinds of fiscal policies (a wasteful public spending and both inter-generational and intra-generational income transfers) on output and welfare by using a non-Walrasian fixed price overlapping generations model with different groups of households concerning the consumption propensity. Among the results derived in this paper, the result of Case 1 (b) that an inter-generational transfer from the young to the old can improve the consumption of the old without harming that of the young seems to be most interesting and important, because it shows that demand stimulation policies can be effective in a Keynesian under-employment economy. We can easily confirm that this result holds true even when we introduce a certain kind of bequest motive or the money in the utility function concerning the preference of households, which means that the result is robust to some extent.

In this paper we consider a “pure” fixed price model where both price and wage are fixed, which corresponds to the simple textbook multiplier model. By introducing the endogenous nominal price determination, the model can be extended to the so called Keynesian *AD-AS* model. To investigate the welfare effects of fiscal policies in such a model is one of the important questions for future researches.

References

Barro, R. and H. Grossman (1971) "A General Disequilibrium Model of Income and Employment", *American Economic Review*, Vol.61, pp82-93

———— (1976) *Money, Employment and Inflation*, Cambridge University Press

Benassy, J. P. (1986) *Macroeconomics: An introduction to the non-Walrasian Approach*, London, Academic Press

———— (2002) *The Macroeconomics of Imperfect Competition and Nonclearing Market*, The MIT Press

Clower, R.W. (1965) "*The Keynesian Counter-Revolution: A Theoretical Appraisal*", in F. Hahn and F. Brechling(eds.), *The Theory of Interest Rates*, London: Macmillan

Malinvaud, E. (1977) *The Theory of Unemployment Reconsidered*, Oxford, Basil Blackwell

Ogawa, T. and Y. Ono (2010) "Public Debt Places No Burden on Future Generations under Demand Shortage", Osaka University GCOE Discussion Paper No.155

Ono, Y. (2011) "Keynesian Multiplier Effect Reconsidered", *Journal of Money, Credit and Banking*, Vol.43, pp787-794

Rankin, N. (1986) "Debt Policy under Fixed and Flexible Prices", *Oxford Economic Papers*, Vol.38, pp481-500

Tanaka, J. (2008) "On the Burden of National Debt in an Underemployment Economy Ⅰ, Ⅱ", The University of Kitakyusyu Working Paper Series