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A Note on "Raising the Mandatory Retirement Age and Its Effect on Long-run Income and Pay As You Go (PAYG) Pensions"

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Abstract

Fanti (2014) showed that raising the mandatory retirement age always reduces capital accumulation and may lower per young income and pension benefit, under the assumption that old and young labor is homogenous (namely, perfect substitutes). However, empirical studies cast doubt on this assumption. Thus, in this paper, we reexamine his analysis by assuming that the two labors are heterogeneous (namely, imperfect substitutes), and prove that his results no longer hold when the elasticity of substitution is not sufficiently high.

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1. Introduction

Faced with rapid population aging and resulting fiscal pressure on the social security system, many developed countries seek to raise both the eligibility age of pension benefit and the mandatory retirement age. Generally, these policies are considered unavoidable in order to mitigate the slowdown of economic growth caused by the rapid drop in the production-age population and to strengthen the sustainability of the social security system.

However, is such a conventional view really right? Fanti (2014) theoretically investigated this question by using a simple overlapping generations model, where the old households allocated a part of their endowed time to the labor supply, and demonstrated that raising the mandatory retirement age always reduces capital accumulation and lowers per young income and pension benefit when the capital share is sufficiently high. If this result is valid, such a policy is harmful to both economic growth and fiscal sustainability of the pension system, contrary to the conventional view. The purpose of this paper is to reexamine his result.

Fanti (2014) derived the result by assuming that old and young labor is homogenous (namely, perfect substitutes). However, a considerable amount of empirical research casts doubt on this assumption. Gruber et.al (2010) investigated the relation between the labor force participation of the old and the young in twelve OECD countries and showed that the labor participation of the young is not negatively but rather positively associated with that of the old. The same conclusions are found in the subsequent researches by Zhang (2012), Munnel and Wu (2012), and Kondo (2016), who respectively studied the cases of China, USA, and Japan¹. These results imply that it is more

¹ Certainly, not all empirical studies support such a result. Martins et.al (2009) showed

realistic to assume that old and young labor is heterogeneous (namely, imperfect substitutes).

Taking these empirical evidences into account, we reexamine Fanti's (2014) analysis by adopting the more general CES technology for the two inputs (old labor and young labor). Under such a specification, Fanti's (2014) analysis corresponds to a special case where its elasticity of substitution is infinite. It shows that the results derived by Fanti (2014) are valid only when the elasticity of substitution is sufficiently high. In other words, if it is not sufficiently high, which seems likely, raising the mandatory retirement age is a proper policy because it stimulates capital accumulation and raises per-young income and pension benefit.

2. Model and Result

The model examined here is almost the same as that used by Fanti (2014) except that the more general production function is assumed, and thus, we describe the structure of the model only briefly.

As for the household sector, the individuals who are born at time t are homogenous and its population (N_t) grows at a constant rate of n. The utility maximization problem of the individual is formulated as follows:

$$\max_{c_t^{\gamma}, c_{t+1}^{o}} U_t = \ln c_t^{\gamma} + \gamma \ln c_{t+1}^{o}$$

s.t.
$$c_t^{\gamma} + s_t = (1 - \tau) w_t^{\gamma}, c_{t+1}^o = R_{t+1} s_t + (1 - \tau) w_{t+1}^o \lambda + z_{t+1} (1 - \lambda)$$

where c_t^{y} is the young-aged consumption, c_{t+1}^{o} is the old-aged consumption, s_t is the

that in Portugal, firms employing older female workers significantly reduced the hiring of younger female workers by a gradual increase in the legal retirement age of female workers. Vestad (2013) found almost one-to-one replacement for retired elderly workers and newly hired young workers in Norway.

savings, $0 < \gamma < 1$ is the subjective discount factor, w_t^{γ} is the young worker's real wage, w_{t+1}^o is the old worker's real wage, τ is the social security tax rate, and $0 \le \lambda \le 1$ is a fraction of time devoted to the labor supply at the old period. Note that in our model, an old worker's real wage is not equal to a young worker's wage because the two workers are assumed to be heterogeneous (namely, imperfect substitutes).

By solving the above problem, we have the following savings function:

$$s_t = \frac{\gamma}{1+\gamma} (1-\tau) w_t^{\gamma} - \frac{1}{1+\gamma} \frac{(1-\tau) w_{t+1}^o \lambda + z_{t+1} (1-\lambda)}{R_{t+1}}.$$
 (1)

As for the public sector, the government runs the Pay As You Go (PAYG) pension system, and the budget constraint at time t is given by

$$z_t(1-\lambda)N_{t-1} = \tau w_t^{\mathcal{Y}} N_t + \tau w_t^o \lambda N_{t-1}, \qquad (2)$$

where z_t is the pension benefit. The left-hand side of (2) represents the social security expenditure and the right-hand side represents the tax receipts.

As for the production sector, firms are supposed to be identical and act competitively. We assume the following production function:

$$Y_t = K_t^a \left[b \left(L_t^y \right)^{\rho} + (1 - b) \left(L_t^o \right)^{\rho} \right]^{\frac{1 - a}{\rho}} (-\infty < \rho \le 1),$$
(3)

where Y_t is the output, K_t is the capital, L_t^y is the young labor, and L_t^o is the old labor. The elasticity of substitution between L_t^y and L_t^o is $\sigma = 1/(1-\rho)$. If $\rho = \infty$ (or $\sigma = 1$) holds, L_t^y and L_t^o are perfect substitutes, which correspond to the case examined by Fanti (2014). If $\rho = 0$ (or $\sigma = 1$) holds, the production function reduces to the Cobb–Douglas type function, i.e., $Y_t = K_t^a L_{1,t}^{b(1-a)} L_{2,t}^{(1-b)(1-a)}$. Under the assumption that capital fully depreciates at the end of each period, the first order conditions of profit maximization are as follows:

$$R_t = a \left(\frac{K_t}{N_t}\right)^{a-1} \left[b \left(\frac{L_t^y}{N_t}\right)^{\rho} + (1-b) \left(\frac{L_t^o}{N_t}\right)^{\rho} \right]^{\frac{1-a}{\rho}},\tag{4.a}$$

$$w_t^{\gamma} = b(1-a) \left(\frac{K_t}{N_t}\right)^a \left(\frac{L_t^{\gamma}}{N_t}\right)^{\rho-1} \left[b \left(\frac{L_t^{\gamma}}{N_t}\right)^{\rho} + (1-b) \left(\frac{L_t^{\rho}}{N_t}\right)^{\rho} \right]^{\frac{1-a}{\rho}-1},$$
(4.b)

$$w_t^{o} = (1-b)(1-a)\left(\frac{K_t}{N_t}\right)^a \left(\frac{L_t^{o}}{N_t}\right)^{\rho-1} \left[b\left(\frac{L_t^{y}}{N_t}\right)^{\rho} + (1-b)\left(\frac{L_t^{o}}{N_t}\right)^{\rho}\right]^{\frac{1-a}{\rho}-1},$$
(4.c)

where R_t is the gross interest rate.

Having formulated the behaviors of individuals, the government, and firms, we can derive the equilibrium dynamics of the economy. The equilibrium conditions of the labor markets (young and old) are respectively given as follows:

$$N_t = L_t^y, \quad \lambda N_{t-1} = L_t^o. \tag{5}$$

Substituting (5) and $N_{t+1} = (1 + n)N_t$ into (4.a), (4.b), and (4.c), we get the following:

$$R_t = aB(\lambda)^{\frac{1-a}{\rho}} k_t^{a-1}, \tag{6.a}$$

$$w_{1,t} = b(1-a)B(\lambda)^{\frac{1-a}{\rho}-1}k_t^a,$$
(6.b)

$$w_{2,t} = (1-b)(1-a)\left(\frac{\lambda}{1+n}\right)^{\rho-1} B(\lambda)^{\frac{1-a}{\rho}-1}k_t^a, \qquad (6.c)$$
$$\left(\text{where } B(\lambda) = b + (1-b)\left(\frac{\lambda}{1+n}\right)^{\rho}, k_t = \frac{K_t}{N_t}\right).$$

The equilibrium condition of the capital market is expressed by the equation: $(1 + n)k_{t+1} = s_t$. Substituting (1), (2), (6.a), (6.b), and (6.c) into this condition and arranging, we can derive the dynamics of capital accumulation:

$$k_{t+1} = \frac{A_1 B(\lambda)^{\frac{1-a}{\rho}}}{A_2 B - A_3} k_t^a,$$
(7)
where $A_1 = \frac{b(1-a)(1-\tau)\gamma}{(1+n)(1+\gamma)}, A_2 = 1 + \frac{1-a}{a(1+\gamma)}, A_3 = \frac{(1-a)b(1-\tau)}{a(1+\gamma)}$.

From (7) we can easily confirm that the steady state exists uniquely and is globally

stable. The steady state level of per young capital is

$$k^{*} = \left[\frac{A_{1}B(\lambda)^{\frac{1-a}{\rho}}}{A_{2}B(\lambda) - A_{3}}\right]^{\frac{1}{1-a}}.$$
(8)

In the steady state, what effects does a rise in λ (the mandatory retirement age) have on per young capital (k^*) , income (y^*) , and pension benefit (z^*) ? Concerning the effect on k^* , we have

$$\frac{\partial k^*}{\partial \lambda} \gtrless 0 \text{ when } \Omega_1 = B'(\lambda) \left[\frac{1}{\rho} \left(\frac{A_2 B - A_3}{A_2 B} \right) - \frac{1}{1 - a} \right] \gtrless 0.$$
(9)

It can be easily confirmed that $\Omega_1 > 0$ holds in the case of $\rho \leq 0$, which means that in such a case $\partial k^* / \partial \lambda > 0$ always holds. In the case of $\rho > 0$, on the other hand, the condition $\Omega_1 \gtrless 0$ can be rewritten as follows:

$$\rho \leq \rho^* \left(\rho^* = (1 - a) \frac{A_2 B(\lambda) - A_3}{A_2 B(\lambda)}, 0 < \rho^* < 1 \right).$$
(10)

Thus, we have the following result:

Result 1: When ρ (the elasticity of substitution between young labor and old labor) is smaller than the critical value ρ^* ($0 < \rho^* < 1$), raising the mandatory retirement age has a positive impact on per young capital.

Figure 1 below illustrates this result. From (10), we can see that a rise in λ lowers k^* only when ρ (the elasticity of substitution between young labor and old labor) is higher than the critical value ρ^* ($0 < \rho^* < 1$). Therefore, if the two labors are perfect substitutes ($\rho = 1$), which correspond to the case investigated by Fanti (2014), a rise in λ has a negative impact on k^* . However, the opposite result holds if ρ is not so high. For example, in the case of $\rho = 0$ (Cobb-Douglas production function), such a change

positively affects k^* .

When ρ is relatively small, why does a rise in λ have the positive impact on k^* ? The intuitive reason is as follows. From (6.b) and (6.c), we have the following:

$$\frac{(w^{y})^*}{(w^o)^*} = \frac{b}{1-b} \left(\frac{\lambda}{1+n}\right)^{1-\rho}.$$

This means that a rise in λ generally widens the wage gap between the two labors and its extent is larger when ρ is smaller. Namely, when ρ is relatively small, a rise in λ causes a relatively large increase in w^{y} and a relatively large decrease in w^{o} . Such a change in the wage profile induces each individual to save more, which results in a higher per young capital.

(Figure 1 around here)

Next, we discuss the effect of a rise in the mandatory retirement age (λ) on per young income (y^*) . Substituting (8) into (3) and arranging, its steady state level is

$$y^* = B(\lambda)^{\frac{1-a}{\rho}} (k^*)^a = \frac{A_1^{\frac{a}{1-a}} B(\lambda)^{\frac{1}{\rho}}}{(A_2 B(\lambda) - A_3)^{\frac{a}{1-a}}}.$$
(11)

After some calculations, we obtain the following:

$$\frac{\partial y^*}{\partial \lambda} \gtrless 0 \text{ when } \Omega_2 = B'(\lambda) \left[\frac{1}{\rho} \left(\frac{A_2 B - A_3}{A_2 B} \right) - \frac{a}{1 - a} \right] \gtrless 0.$$
(12)

It can be easily confirmed that $\Omega_2 > 0$ holds in the case of $\rho \leq 0$, which means that in such a case, $\partial y^* / \partial \lambda > 0$ always holds, whereas in the case of $\rho > 0$, the condition $\Omega_2 \geq 0$ can be rewritten as follows:

$$\rho \leq \frac{\rho^*}{a},\tag{13}$$

where ρ^* is the critical value appeared in (10). Accordingly, the following result holds.

Result 2: When ρ (the elasticity of substitution) is smaller than the critical value ρ^*/a (> ρ^*), raising the mandatory retirement age has a positive impact on per young income.

This result means that a rise in λ lowers y^* only when ρ (the elasticity of substitution) is higher than the critical value ρ^*/a , which is a larger value than ρ^* because its denominator is the capital share (0 < a < 1). Therefore, if the capital share is sufficiently large and accordingly $\rho^*/a < 1$ holds, a rise in λ lowers y^* in the case of $\rho = 1$, as shown by Fanti (2014)². Figure 2 illustrates this case. However, when ρ is smaller than ρ^*/a , the opposite result holds. Furthermore, as depicted in Figure 2, the range of ρ under which $\partial y^*/\partial \lambda > 0$ holds becomes broader than the range under which $\partial k^*/\partial \lambda > 0$ holds. The intuitive reason is as follows. From (11), a rise in λ affects y^* through two channels: per young capital (k^*) and elderly labor supply ($B(\lambda)$). As the latter effect is always positive, $\partial y^*/\partial \lambda > 0$ holds.

(Figure 2 around here)

Finally, we discuss the effect of a rise in the mandatory retirement age (λ) on per-young pension benefit (z^*) . From (2), (6.b), (6.c), and (8), its steady state level can be derived as follows:

² On the contrary, if the capital share is not sufficiently large and therefore $\rho^*/a > 1$ holds, a rise in λ always has a positive impact on y^* .

$$z^{*} = \frac{\tau(1+n)(w^{y})^{*} + \tau\lambda(w^{o})^{*}}{1-\lambda} = (1+n)(1-a)\frac{y^{*}}{1-\lambda}$$
(14)
$$= \frac{A_{4}B(\lambda)^{\frac{1}{\rho}}}{(1-\lambda)(A_{2}B(\lambda) - A_{3})^{\frac{a}{1-a}}} \left(A_{4} = \tau(1+n)(1-a)A_{1}^{\frac{a}{1-a}}\right).$$

After some calculations, we have

$$\frac{\partial z^*}{\partial \lambda} \gtrless 0 \text{ when } \Omega_3 = (1-\lambda)B'(\lambda) \left[\frac{1}{\rho} \left(\frac{A_2 B - A_3}{A_2 B} \right) - \frac{a}{1-a} \right] + B(\lambda) \left(\frac{A_2 B - A_3}{A_2 B} \right) \gtrless 0 \tag{15}$$

It can be easily confirmed that $\Omega_3 > 0$ holds in the case of $\rho \leq 0$, which means that in such a case, $\partial z^* / \partial \lambda > 0$ always holds. We can also show that $\Omega_3 > 0$ always holds, even in the case of $\rho > 0$ if the following condition is satisfied.

$$\rho^*/a \ge (1-\lambda)B'(\lambda)/B \tag{16}$$

Here, ρ^*/a is the critical value appeared in (13). On the other hand, if (16) is not satisfied, the condition $\Omega_3 \gtrless 0$ can be rewritten as follows:

$$\rho \leq \rho^{**} \left(\rho^{**} = \frac{\frac{\rho^*}{a}}{1 - \frac{B(\lambda)}{(1 - \lambda)B'(\lambda)} \frac{\rho^*}{a}} \left(> \frac{\rho^*}{a} \right) \right).$$
(17)

Summarizing these points, we have the following result.

Result 3: If (16) is satisfied, raising the mandatory retirement age always has a positive impact on per young pension benefit. Even if (16) is not satisfied, when ρ (the elasticity of substitution) is smaller than the critical value $\rho^{**}(>\rho^*/a>\rho^*)$, the same result holds.

From Result 3, we can see that a rise in λ lowers z^* only when (16) is not satisfied and ρ is higher than ρ^{**} . Therefore, if $\rho^{**} < 1$ holds, raising the mandatory retirement age has a negative impact on z^* (namely, it worsens the sustainability of pension system) in the case of $\rho = 1$, as pointed out by Fanti (2014)³. Figure 3 illustrates this case. However, when ρ is smaller than ρ^{**} , the opposite result holds. As depicted in Figure 3, the range of ρ under which $\partial z^*/\partial \lambda > 0$ holds becomes broader than the range under which $\partial y^*/\partial \lambda > 0$ holds. The intuitive reason is as clear. As $z^* = (1+n)(1-a)\frac{y^*}{1-\lambda}$ holds from (14), a rise in λ affects z^* through two channels: the numerator (y^*) and the denominator $(1-\lambda)$. As the latter effect is always positive, $\partial z^*/\partial \lambda > 0$ holds in the broader range of ρ than the range under which $\partial y^*/\partial \lambda > 0$ holds.

(Figure 3 around here)

3. Conclusion

Fanti (2014) demonstrated that raising the mandatory retirement age always reduces capital accumulation and may lower per young income and pension benefit under the assumption that young labor and old labor are perfect substitutes. However, we proved that the opposite result holds in the more realistic assumption that the two labors are imperfect substitutes. Our result indicates that the conventional view that raising both the eligibility age of pension benefit and the mandatory retirement age is necessary is proper.

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³ On the other hand, if $\rho^{**} > 1$ holds, a rise in λ always has a positive impact on z^* .

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(Figure 1: The effect of a rise in the mandatory retirement age (λ) on per young capital (k^*))



(Figure 2: The effect of a rise in the mandatory retirement age (λ) on per young income (y^*))

$$\rho = 0$$
 $\rho = \rho^*$ $\rho = \rho^*/a$ $\rho = \rho^{**}$ $\rho = 1$



(Figure 3: The effect of a rise in the mandatory retirement age (λ) on per young pension benefit (z^*))