Theoretical Analysis of a Dual Labor Market

in Monopolistic Competition

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Abstract

We study the characteristics of a dual labor market economy comprising the primary sector (or regular employment sector), where wages are determined through bargaining between firms and labor unions, and the secondary sector (or non-regular employment sector), where the wage is determined competitively, using a static general equilibrium model with monopolistic competition. We show how changes in unions' bargaining power, the labor productivity differential, and the firm's monopoly power (or the degree of competitiveness in the goods market) affect the inter-sectoral wage and employment ratios, the labor distribution rates in both sectors, output, price, and the welfare of regular and non-regular workers. We also compare the results of two alternative bargaining systems, namely the "right to manage" model and the "efficient bargaining" model.

Keywords : dual labor market, regular and non-regular employment, trade union, monopolistic competition

JEL Classification : E10, E24, J01

1. Introduction

In Japan, the share of non-regular workers as a proportion of the total labor force has been increasing over the past 30 years. According to Roudou-Keizai-Hakusyo (Labor Economic White Paper) published in 2012, the ratio of non-regular workers that stood at approximately 15% in 1985 had risen to 35% in 2012. It also points out that the income gap between regular and non-regular workers is striking and seen as one of the main reasons for the growing income inequality in Japan in recent years. Thus, the Japanese economy can be characterized as a dual labor market economy.

The purpose of this study is to explore theoretically the macroeconomic characteristics of a dual labor market economy that comprises regular employment with relatively high wages and non-regular employment with relatively low wages. In particular, using a static general equilibrium model with monopolistic competition, we construct a dual labor market model where the wage of the primary sector (regular employment sector) is determined through bargaining between firms and labor unions and the wage of the secondary sector (non-regular employment sector) is determined competitively. We investigate how changes in the labor unions' bargaining power, the labor productivity differential, and the firm's monopoly power (or the degree of competitiveness in the goods market) affect the inter-sectoral wage and employment ratios, the labor distribution rates in both sectors, output, price, and the welfare of regular and non-regular workers.

There have been a considerable amount of studies on the dual labor market, roughly classified as those with models where the labor union determines the wage (and employment) in the primary sector (McDonald and Solow (1985)) and those where the determination of wage is based on the efficiency wage hypothesis (Bullow and Summers (1986), Jones (1987), and Saint-Paul (1997)). This study belongs to the former classification.

McDonald and Solow (1985) developed a dual labor market model to explain the fact that in the United States, during business cycles, the adjustment of wages (employment) is relatively larger (resp. smaller) in the primary sector comprising large established firms. In it, wage and employment in the primary sector are determined by "efficient bargaining (EB),"[1] while they are determined competitively in the secondary sector. The model investigates the effect of a "demand shock" on wage and employment in both sectors.

Since their argument was based on the partial equilibrium analysis wherein attention

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[1] "Efficient bargaining" is a type of bargaining process wherein both wage and employment are determined. It was first described by McDonald and Solow (1981).

is focused mainly on the labor market, some authors have attempted to extend it to the general equilibrium analysis. For example, Sanner (2006) extended the model of Blanchard and Giavazzi (2003), which studied the effects of (de)regulation in both goods and labor markets in a static general equilibrium model with monopolistic competition and labor union, to a dual labor market model. Using numerical analysis, they examined whether the union wage gap (namely, the wage differentials between union members and the rest), empirically estimated by Blanchflower (1996), could be replicated in the model. By extending a dual labor market model of the type described by McDonald and Solow (1985) to a simple endogenous growth model with monopolistic competition, de Groot (2001) investigated the relationship between "wait unemployment" in the secondary sector and economic growth caused by R&D activity in the primary sector. These studies are similar to ours in that they all adopt a general equilibrium framework with monopolistic competition; however, the purpose of the analysis is somewhat different. These studies also differ from ours in that they formulate a dual labor market economy as two different industries (the sector of large firms characterized by monopolistic competition and the sector of small firms characterized by perfect competition), while we formulate it as two different classes of workers (regular and non-regular) employed by the representative firm.

Osumi (1998) and Nakatani (2004) are similar to our model with respect to the formulation of a dual labor market mentioned above. Osumi (1998) introduced heterogenous workers (skilled workers who are complementary with physical capital and unskilled workers who are not) into a macroeconomic model of the McDonald and Solow (1985) type. He showed that wage adjustment (employment) during business cycles is relatively larger (smaller) in the *secondary* sector, which is the opposite of the result found by McDonald and Solow (1985). Nakatani (2004) presented a simple macroeconomic model where a representative monopolistically competitive firm employs both regular workers whose wage is determined through bargaining and non-regular workers whose wage is determined competitively and investigated the effects of changes in the unions' bargaining power, the labor productivities of both types of workers, and the firm's monopoly power on wages, employment, labor distribution, and output.

The aim of our study is similar as that of Nakatani (2004). However, he adopted a rather simple macroeconomic model without a microeconomic foundation, leaving room to refine and improve the model. Thus, by extending the static general equilibrium model with monopolistic competition and labor unions[2] of Dutt and Sen (1997) to a dual labor

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[2] The basic structure of the Dutt and Sen (1997) model is almost the same as that of Blanchard and Giavazzi (2003) as both are the static general equilibrium models with

market model, we re-examine his argument within a more rigorous theoretical framework. Although our results are close to those of Nakatani (2004), there are some differences (which are summarized in section 3). We also investigate aspects not dealt with by him. For example, we compare the results of two bargaining systems (the "right to manage" model and the "efficient bargaining" model), whereas Nakatani considered only the former case. Furthermore, we examine the effects of exogenous shocks on some aspects of the monetary economy and on the individual welfare of both regular and non-regular workers, which his study does not consider.

Our study can also be seen as an extension of Blanchard and Giavazzi (2003) who looked at a single labor market. While they studied the effects of (de)regulation in the goods market (the changes in elasticity of substitution between differentiated goods) and (de)regulation in the labor market (changes in the unions' bargaining power) on real wage and unemployment, our study explores their effects on the inter-sectoral wage and employment ratios, the labor distribution rates, output, price, and individual welfare.

The rest of this paper is organized as follows. In Section 2, we present our dual labor market model and derive our comparative static results (shown by propositions 1～7). In Section 3, we compare these results with those of Nakatani (2004). Section 4 concludes the paper.

2．Analysis

2．1 *Households*

In this subsection, we formulate the behavior of households and then derive the demand for each differentiated good and the real aggregate demand.

Consider a static monopolistic competition model with n types of differentiated consumption goods. n is also the number of firms because the good i ($i = 1, 2, \cdots, n$) is produced monopolistically by the firm i. Since we neglect free entry of potential firms for simplicity, n is an exogenous parameter in this model.

There are N ($> n$) workers and a "capitalist" (or an asset holder) in this model and each household is homogenous with respect to its preferences. Each worker supplies one unit of labor inelastically, expends a part of her wage income to purchase a differentiated good, and the rest is held as money demand. The labor market comprises the primary sector where regular workers are employed and the secondary sector where non-regular

monopolistic competition and labor unions. However, the purpose of the analyses are totally different. The former's purpose is to provide a microeconomic foundation for the Kalecki (1971) model, while the latter's is to study the effects of (de)regulation in goods and labor markets.

workers are employed. The nominal wage of the primary sector W_1 is determined through bargaining between firms and labor unions,[3] while that of the secondary sector W_2 is determined competitively. Since $W_1 > W_2$ holds as the result of bargaining (which will be demonstrated later), every worker hopes to work in the primary sector. However, since the number of employees in the primary sector L_1 is assumed to be smaller than N (the total number of workers), unemployed workers move to the secondary sector and find a job there. Accordingly, there is no unemployment in our model, and $L_1 + L_2 = N$ holds (where L_2 is the number of employees in the secondary sector). In contrast, a capitalist does not supply labor because she is endowed with money stock and further receives profits as an owner of all firms. She spends a part of it to purchase differentiated goods and the rest is held as money demand. Therefore, the utility maximization problem of each household can be formulated as

$$(1) \qquad \max_{c_{ij}, M_j^d} \quad U_j = (C_j)^\alpha \left(\frac{M_j^d}{P} \right)^{1-\alpha}. \qquad \left(C_j \equiv n \left[\sum_{i=1}^n \frac{1}{n} (c_{ij})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right)$$

$$\text{s.t.} \quad \sum_{i=1}^n p_i c_{ij} + M_j^d = I_j. \qquad (I_1 = W_1, \quad I_2 = W_2, \quad I_3 = \Pi + M)$$

Here, the subscript j refers to the "class" of each household; namely, $j = 1$ ($j = 2$) refers to an employee in the primary (secondary) sector and $j = 3$ refers to a capitalist. c_{ij} is household j's consumption of good i; C_j is household j's sub-utility defined in (1); η is the elasticity of substitution between goods; p_i is the nominal price of the good i; P is the aggregate price index (defined in (3)); M_j^d is household j's nominal money demand; M is the initial money stock held by a capitalist; and Π is the sum of nominal profits of all firms (that is, $\Pi \equiv \sum_{i=1}^n \pi_i$). With respect to the elasticity of substitution η, $\eta > 1$ is assumed to assure a positive markup rate in equilibrium. Note that in the case of $\eta \to \infty$, the perfect substitution between goods holds and the goods market becomes perfectly competitive. Thus, η can be interpreted as the degree of competitiveness in the goods market.

Problem (1) can be solved as follows: We first derive the demand for each differentiated

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[3] To be precise, the nominal wages in the primary sector should be expressed as W_{i1} ($i = 1, 2, \cdots, n$) because the negotiations are conducted at the firm level. However, since they do not depend on i in equilibrium, we use this notation for simplicity.

good that minimizes the expenditure under a given level of sub-utility.

$$(2) \qquad \min_{c_{ij}} \sum_{i=1}^{n} p_i c_{ij} \qquad \text{s.t.} \qquad C_j = n \left[\sum_{i=1}^{n} \frac{1}{n} (c_{ij})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad (C_j : \text{given})$$

Solving this, we can derive household j's optimal demand for good i as

$$(3) \qquad c_{ij} = \left(\frac{p_i}{P} \right)^{-\eta} \frac{C_j}{n}, \qquad \text{where } P \equiv \left[\frac{1}{n} \sum_{i=1}^{n} (p_i)^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

Here, the aggregate price index P defined above refers to the minimum cost needed to raise sub-utility by one unit. From (3), we see that the optimal demand for good i decreases as its price increases, and increases as the given sub-utility increases. Solving (2), we obtain

$$(4) \qquad \sum_{i=1}^{n} p_i c_{ij} = P C_j.$$

This shows that the sub-utility C_j can be interpreted as household j's real expenditure level. Summing (3) with respect to j, we obtain the aggregate demand for good i as

$$(5) \qquad y_i \; (\equiv \sum_{j=1,2,3} c_{ij}) = \left(\frac{p_i}{P} \right)^{-\eta} \frac{C}{n}, \qquad \text{where } C \equiv C_1 L_1 + C_2 L_2 + C_3.$$

Substituting (4) into the original problem (1)'s budget constraint, we arrive at the second step problem.

$$(6) \qquad \max_{C_j, M_j^d} \; U_j = (C_j)^{\alpha} \left(\frac{M_j^d}{P} \right)^{1-\alpha} \qquad \text{s.t.} \quad P C_j + M_j^d = I_j$$

Solving this, we have household j's optimal nominal expenditure and money demand.

$$(7) \qquad P C_j = \alpha I_j, \quad M_j^d = (1-\alpha) I_j$$

From (1) and (7), household j's real expenditures ($j = 1,2,3$) are, respectively,

$$(8) \qquad C_1 = \frac{\alpha W_1}{P}, \quad C_2 = \frac{\alpha W_2}{P}, \quad \text{and } C_3 = \frac{\alpha(\Pi + M)}{P}.$$

Accordingly, aggregated real expenditure C defined by (5) can be expressed as

$$(9) \qquad C = \frac{\alpha}{P} [W_1 L_1 + W_2 L_2 + \Pi + M].$$

2.2　Firms and Labor Unions

In this subsection, we formulate the behaviors of firms and labor unions and then investigate the properties of the wage and employment ratios between primary and secondary sectors and the labor distribution rates in both sectors.

We assume firm-level negotiations (namely, decentralized bargaining). Furthermore, we detail the "right-to-manage" (hereafter, RTM) model where only the nominal wage is bargained and the employment level is determined by the firm after negotiations. The case of the other bargaining system, i.e., the "efficient bargaining" (hereafter, EB) model, will be studied in 2.4.

Here, we first discuss the firm's profit maximization problem under a given wage, and then formulate the bargaining problem of the RTM type.

2.2.1 The firm's profit-maximization problem

Firm i supplies good i monopolistically. Its nominal profit is

(10)　　　$\pi_i = p_i y_i - (W_{i1} l_{i1} + W_2 l_{i2})$,

where y_i is firm i's output and W_{i1} is the nominal wage that firm i pays to employees in the primary sector, which is determined through bargaining with a labor union. Firm i's production function is characterized by constant elasticity of substitution (hereafter, CES) technology:

(11)　　　$y_i = [a(A_1 l_{i1})^\rho + (1-a)(A_2 l_{i2})^\rho]^{1/\rho}$,　　　$(-\infty < \rho < 1)$

where the elasticity of substitution between two types of labor (σ) is given by $\sigma = 1/(1-\rho)$. In the case of $\rho \to 0$ $(\sigma \to 1)$, this production function approaches the Cobb–Douglas technology: $y_i = (A_1 l_{i1})^a (A_2 l_{i2})^{1-a}$.

Firm i's profit maximization problem can be solved as follows: We first derive the conditional labor demands in the primary and secondary sectors that minimize the production cost for a given level of output y_i.

(12)　　　$\min_{l_{i1}, l_{i2}} W_{i1} l_{i1} + W_2 l_{i2}$　　　s.t.　(11)　　　(y_i is given)

The first-order condition of (12) is

(13)　　　$\dfrac{l_{i1}}{l_{i2}} = \left(\dfrac{a}{1-a}\right)^{\frac{1}{1-\rho}} \left(\dfrac{A_1}{A_2}\right)^{\frac{\rho}{1-\rho}} \left(\dfrac{W_{i1}}{W_2}\right)^{-\frac{1}{1-\rho}}$.

From (13), we can confirm that the optimal employment ratio l_{i1}/l_{i2} decreases as the wage ratio (W_{i1}/W_2) increases, and increases as the labor productivity ratio A_1/A_2

increases. The reason for the latter is that a change of A_1/A_2 affects the marginal rate of technical substitution (or the slope of an isoquant).

From (11), (13), and (10) we can derive the conditional labor demands and the profit function, respectively.

$$(14) \qquad l_{i1} = \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1/\rho} \frac{y_i}{A_1} \qquad \left(X \equiv \left(\frac{a}{1-a} \frac{A_1}{A_2} \right)^{-1} \frac{W_{i1}}{W_2} \right)$$

$$(15) \qquad l_{i2} = X^{\frac{1}{1-\rho}} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1/\rho} \frac{y_i}{A_2}$$

$$(16) \qquad \pi_i = p_i y_i - \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} \frac{W_{i1}}{aA_1} y_i$$

In the second step, firm i determines the price of good i that maximizes the profit function (16) subject to the aggregate demand for good i (shown by (5)).

$$(17) \qquad \max_{p_i} \ (16) \qquad \text{s.t.} \ (5)$$

Solving this, the optimal price and the maximized nominal profit are

$$(18) \qquad p_i = \frac{\eta}{\eta - 1} \frac{W_{i1}}{aA_1} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1-\rho}{\rho}},$$

$$(19) \qquad \pi_i = K_0 W_{i1}^{1-\eta} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{\frac{(1-\rho)(\eta-1)}{\rho}}. \qquad (K_0 : \text{a constant})$$

2.2.2 Bargaining between a firm and a labor union

The negotiations between a firm and a labor union are conducted considering the firm's profit maximization behavior mentioned above. Following Dutt and Sen (1997), N workers are partitioned into n parts and each group (with $\mu = N/n$ members) forms a labor union. The negotiation is conducted at the firm level, that is, in a decentralized manner. The objective of a labor union is to maximize the expected income V_i of its workers:

$$V_i = \frac{l_{i1}}{\mu} \times W_{i1} + \left(1 - \frac{l_{i1}}{\mu} \right) \times W_2.$$

Assuming the standard Nash bargaining, the nominal wage in the primary sector W_{i1} is determined such that it maximizes the Nash product:

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$$Z_i = (\pi_i - \pi_i^0)^{1-\beta}(V_i - V_i^0)^{\beta}, \qquad (0 < \beta < 1)$$

where π_i is firm i's nominal profit; π_i^0 is its reservation profit, which is supposed to be 0, V_i^0 is the reservation wage for workers which is supposed to be equal to the nominal wage in the secondary sector $(V_i^0 = W_2)$, and β is the union's bargaining power. From these, the Nash product can be arranged by

(20)
$$Z_i = (\pi_i)^{1-\beta}\left[\frac{l_{i1}}{\mu}(W_{i1} - W_2)\right]^{\beta}.$$

Thus, the bargaining problem between a firm and a labor union can be formulated as

(21)
$$\max_{W_{i1}} \ (20) \qquad \text{s.t.} \quad (19), \quad (14), \quad (5), \quad \text{and} \quad (18).$$

A. Determination of the inter-sectoral wage ratio

We can derive the equilibrium inter-sectoral wage ratio $k \ (\equiv W_1/W_2)$ by solving problem (21). Arranging the first order condition $(\partial Z_i/\partial W_{i1} = 0)$, we have the following equation for k:

(22)
$$\frac{(1-a)\beta}{1-\rho}\left(\frac{a}{1-a}\frac{A_1}{A_2}\right)^{-\frac{\rho}{1-\rho}} k^{\frac{\rho}{1-\rho}}(1-\rho k) = a(\eta-1)k - a[\eta-(1-\beta)]$$

From this, we can see that W_{i1}, which maximizes (20), does not depend on the index i and all firms set the same wage.

(23)
$$W_{i1} = W_1$$

Defining the left hand side of (22) as $F^L(k)$, it has the following properties.

(24)
$$F^L(k) \equiv K_1 \, k^{\frac{\rho}{1-\rho}}(1-\rho k) \qquad (K_1: \text{a constant})$$

$$F^L(0) = 0, \qquad \frac{dF^L(k)}{dk} = \frac{\rho}{1-\rho}\, k^{\frac{\rho}{1-\rho}-1}(1-k), \qquad \left.\frac{dF^L(k)}{dk}\right|_{k=1} = 0$$

Here, we make the following assumption concerning the range of the elasticity of substitution.

(25) $0 < \rho < \dfrac{\eta - 1}{\eta - (1 - \beta)} (<1)$ or equivalently, $1 < \sigma < 1 + \dfrac{\eta - 1}{\beta}$

Since the Cobb–Douglas technology corresponds to the case of $\rho = 0$ or $\sigma = 1$, assumption (25) means that we focus on the case where the elasticity of substitution is a little higher than that suggested by Cobb–Douglas technology.[4,5]

(Figure 1 around here)

Figure 1 depicts (22) under assumption (25). It shows that there exists a unique inter-sectoral wage ratio k that is larger than 1. In this case, the following holds:

$$(\eta - 1)k - [\eta - (1 - \beta)] > 0, \quad 1 - \rho k > 0$$

A comparative static analysis of (22) that considers the above yields the following:[6]

(26) $\dfrac{\partial k}{\partial \beta} > 0, \quad \dfrac{\partial k}{\partial (A_1 / A_2)} < 0, \text{ and } \dfrac{\partial k}{\partial \eta} < 0$

Proposition 1 (properties of the equilibrium inter-sectoral wage ratio)

Under assumption (25), the equilibrium inter-sectoral wage ratio is given by k ($\equiv W_1 / W_2$) that satisfies (22) and has the following properties:

(a) An increase in the unions' bargaining power β raises W_1 / W_2.

(b) An increase in the inter-sectoral labor productivity ratio A_1 / A_2 reduces W_1 / W_2.

(c) An increase in the elasticity of substitution between goods η (namely, an increase in the degree of competitiveness in the goods market) reduces W_1 / W_2.

Proposition 1 holds because of the following reasons: First, an increase in β raises W_1 / W_2 because a higher W_1 is set by bargaining aimed at raising labor's share. Second, an increase in A_1 / A_2 reduces W_1 / W_2 because it pulls up labor's share disproportionately by, for example, increasing the probability of being employed in the primary sector and accordingly a lower W_1 is set to smooth the shares for both parties. Finally, an increase in η reduces W_1 / W_2 because it reduces the firm's share

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[4] We can similarly investigate the case of $\dfrac{\eta - 1}{\eta - (1 - \beta)} < \rho < 1$ and can derive the different comparative static results from those under (25). In this case, however, the signs of the comparative statics are often indeterminate.

[5] The assumption that the elasticity of substitution between heterogenous labors is larger than 1 is compatible with empirical studies. See, for example, Johnson (1997) for this point.

[6] Proof is available upon request.

disproportionately through a decline in its monopoly power and accordingly a lower W_1 is set to smooth both shares.

In the case of Cobb–Douglas technology ($\rho = 0$ or $\sigma = 1$) the equilibrium inter-sectoral wage ratio k can be calculated explicitly as

$$k = 1 + \frac{\beta}{a(\eta - 1)}.$$

In this case $\partial k / \partial (A_1 / A_2) = 0$ holds, which is different from the result $\partial k / \partial (A_1 / A_2) < 0$ under CES technology. This is because under Cobb–Douglas technology, a change of A_1 / A_2 is equivalent to a change in the total factor productivity (TFP) and it does not lead to any biased changes in shares for both parties. The other results in Proposition 1 remain unchanged.

B. Determination of the inter-sectoral employment ratio

Next, we derive the equilibrium inter-sectoral employment ratio and investigate its properties. From (13) and $k = W_1 / W_2$, we have

(27) $$\frac{L_1}{L_2} = \left(\frac{a}{1-a} \right)^{\frac{1}{1-\rho}} \left(\frac{A_1}{A_2} \right)^{\frac{\rho}{1-\rho}} k^{-\frac{1}{1-\rho}}. \qquad (L_1 \equiv \sum_{i=1}^{n} l_{i1}, \ L_2 \equiv \sum_{i=1}^{n} l_{i2})$$

From this, we see that the equilibrium employment ratio L_1 / L_2 is inversely correlated with the equilibrium wage ratio k as k lowers the relative labor demand under a given isoquant. From (26) and (27) we can derive

$$\frac{\partial (L_1 / L_2)}{\partial \beta} < 0, \ \frac{\partial (L_1 / L_2)}{\partial (A_1 / A_2)} > 0, \text{ and } \frac{\partial (L_1 / L_2)}{\partial \eta} > 0.$$

Proposition 2 (properties of the equilibrium inter-sectoral employment ratio)

Under assumption (25), the equilibrium inter-sectoral employment ratio L_1 / L_2 is given by (27) and has the following properties:

(a) An increase in the unions' bargaining power β reduces L_1 / L_2.

(b) An increase in the inter-sectoral labor productivity ratio A_1 / A_2 raises L_1 / L_2.

(c) An increase in the elasticity of substitution between goods η (namely, an increase in the degree of competitiveness in the goods market) raises L_1 / L_2.

In the case of Cobb–Douglas technology ($\rho = 0$ or $\sigma = 1$), $\partial (L_1 / L_2) / \partial (A_1 / A_2) = 0$ holds, which is different from the result $\partial (L_1 / L_2) / \partial (A_1 / A_2) > 0$ under CES technology. This is because $\partial k / \partial (A_1 / A_2) = 0$ holds under Cobb–Douglas technology as explained above. The other results in Proposition 2 do not change.

C. The determination of the labor distribution rates

Finally, we derive the equilibrium labor distribution rates (namely, the ratio of real wage income over output) in both sectors and investigate their properties.

Since p_i (the good i's price) does not depend on the index i from (18) and (23), we have the following by the definition of the aggregate price index shown in (3):

(28) $\qquad p_i = P$

Substituting this into (5), the output of each firm also becomes equal.

(29) $\qquad y_i = y = C/n$

Considering these, the conditional labor demands and the aggregate price index shown by (14), (15), and (18) can be rewritten as

(30) $\qquad L_1 = \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1}{\rho}} \frac{ny}{A_1}, \qquad \left(X \equiv \left(\frac{a}{1-a} \frac{A_1}{A_2} \right)^{-1} k \right)$

(31) $\qquad L_2 = X^{\frac{1}{1-\rho}} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1}{\rho}} \frac{ny}{A_2},$

(32) $\qquad P = \frac{\eta}{\eta-1} \frac{W_1}{aA_1} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}}.$

Using these, the equilibrium labor distribution rates can be calculated as

(33) $\qquad \Omega_1 \ \left(\equiv \frac{W_1 L_1}{Pny} \right) = \frac{a(\eta-1)}{\eta} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1},$

(34) $\qquad \Omega_2 \ \left(\equiv \frac{W_2 L_2}{Pny} \right) = \frac{(1-a)(\eta-1)}{\eta} X^{\frac{\rho}{1-\rho}} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1}.$

These yield,

$$\frac{\partial \Omega_1}{\partial \beta} < 0, \ \frac{\partial \Omega_1}{\partial (A_1/A_2)} > 0, \ \frac{\partial \Omega_1}{\partial \eta} > 0, \ \frac{\partial \Omega_2}{\partial \beta} > 0, \ \frac{\partial \Omega_2}{\partial (A_1/A_2)} < 0, \ \frac{\partial \Omega_2}{\partial \eta} : \text{ambiguous.}$$

Proposition 3 (properties of the equilibrium labor distribution rates)

Under assumption (25) the equilibrium labor distribution rates in the primary (secondary) sector Ω_1 (Ω_2) are given by (33) and (34), respectively. They have the following properties:

(a) An increase in the union's bargaining power β reduces Ω_1 and raises Ω_2.

(b) An increase in the inter-sectoral labor productivity ratio A_1 / A_2 raises Ω_1 and reduces Ω_2.

(c) An increase in the elasticity of substitution between goods η (namely, an increase in the degree of competitiveness in the goods market) raises Ω_1 but the effect on Ω_2 is ambiguous.

The intuitive reason for the results of Proposition 3 can be found in the results of Proposition 1 and 2. When β increases, the nominal wage in the primary (secondary) sector becomes relatively higher (lower), from Proposition 1, and the employment in the primary (secondary) sector becomes relatively lower (higher), from Proposition 2. In the case where the elasticity of technological substitution σ is larger than 1, the latter effect (which we call the "relative employment effect") exceeds the former effect (which we call the "relative wage effect"), so an increase in β reduces (raises) the labor distribution rate in the primary (secondary) sector. The reason for higher A_1 / A_2 causing higher Ω_1 and lower Ω_2 can be explained in the same way. Finally, an increase in η affects the labor distribution rates through two different channels. One channel is the same as that mentioned above, and it raises (reduces) Ω_1 (Ω_2). The other channel is that it reduces the firm's markup and the profit distribution rate, which has a positive impact on both Ω_1 and Ω_2. Consequently, Ω_1 necessarily becomes higher, while the effect on Ω_2 is unclear because of two contrary effects.

In the case of Cobb–Douglas technology ($\rho = 0$ or $\sigma = 1$), the equilibrium labor distribution rates can be simplified as

$$\Omega_1 \;=\; \frac{a(\eta - 1)}{\eta}, \qquad \Omega_2 \;=\; \frac{(1-a)(\eta - 1)}{\eta}.$$

Thus, we have

$$\frac{\partial \Omega_1}{\partial \beta} = 0, \quad \frac{\partial \Omega_1}{\partial (A_1 / A_2)} = 0, \quad \frac{\partial \Omega_1}{\partial \eta} > 0, \quad \frac{\partial \Omega_2}{\partial \beta} = 0, \quad \frac{\partial \Omega_2}{\partial (A_1 / A_2)} = 0, \text{ and } \frac{\partial \Omega_2}{\partial \eta} > 0.$$

The reason for changes in β and A_1 / A_2 having no impact on Ω_1 and Ω_2 under Cobb–Douglas technology is that the two contrary effects (the "relative wage effect" and the "relative employment effect") cancel out each other because the elasticity of technical substitution σ is equal to 1. With regard to the effect of an increase in η, since the effect through the first channel disappears and only the second channel affects Ω_1 and Ω_2, it increases both.

2.3 Macroeconomic Equilibrium

In this subsection, we study the properties of output, price, and the welfare of primary

and secondary workers in macroeconomic equilibrium. We also examine whether the neutrality of money holds in equilibrium.

A. Properties of output and price

The equilibrium output can be derived from the labor market equilibrium condition in the secondary sector. From (30), the labor supply in the secondary sector L_2^s is

$$L_2^s = N - \left[a + (1-a) X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1}{\rho}} \frac{ny}{A_1} . \qquad (X \equiv \left(\frac{a}{1-a} \frac{A_1}{A_2} \right)^{-1} k)$$

From this and the labor demand in the secondary sector shown by (30), we have

$$(35) \qquad y = \left[\frac{1}{A_1} + \frac{1}{A_2} X^{\frac{1}{1-\rho}} \right]^{-1} \left[a + (1-a) X^{\frac{\rho}{1-\rho}} \right]^{\frac{1}{\rho}} \frac{N}{n} .$$

The equilibrium price can be derived from the equilibrium condition on the real aggregate demand and supply in the goods market. From (9) and (10), the real aggregate demand can be rewritten as

$$(36) \qquad C = \frac{\alpha}{P} [W_1 L_1 + W_2 L_2 + \Pi + M] = \frac{\alpha}{P} [\sum_{i=1}^{n} p_i y_i + M]$$

$$= \frac{\alpha}{P} [nPy + M].$$

From (5) and (28), the real aggregate supply is given by

$$(37) \qquad C = ny .$$

Thus, we have

$$(38) \qquad y = \frac{\alpha}{1-\alpha} \frac{M}{nP} ,$$

which shows that output is inversely correlated with price. This is because a higher price decreases the real aggregate demand through a decline in a capitalist's real money holding (namely, the negative "real balance effect"). From (35) and (38), the equilibrium price can be calculated as

$$(39) \qquad P = \left[\frac{1}{A_1} + \frac{1}{A_2} X^{\frac{1}{1-\rho}} \right] \left[a + (1-a) X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1}{\rho}} \frac{\alpha}{1-\alpha} \frac{M}{N} .$$

Using (35) and (39) we can obtain the following comparative static results. (Note that here we examine the effect of an increase in the primary labor productivity level A_1, not the relative productivity A_1 / A_2.)

$$\frac{\partial y}{\partial \beta} < 0, \quad \frac{\partial y}{\partial A_1} > 0, \quad \frac{\partial y}{\partial \eta} > 0, \quad \frac{\partial P}{\partial \beta} > 0, \quad \frac{\partial P}{\partial A_1} < 0, \text{ and } \frac{\partial P}{\partial \eta} < 0$$

Proposition 4 (properties of the equilibrium output and price)

Under assumption (25), the equilibrium output y and price P are given by (35) and (39), and they have the following properties:

(a) An increase in the unions' bargaining power β reduces y and raises P.

(b) An increase in the primary labor productivity A_1 raises y and reduces P.

(c) An increase in the elasticity of substitution between goods η (namely, an increase in the degree of competitiveness in the goods market) raises y and reduces P.

Proposition 4 can be explained as follows: An increase in β raises the production cost by increasing the nominal wage in the primary sector; this pulls up the price under constant markup. Furthermore, higher prices reduce output because of the negative real balance effect. That is, a higher β generates "cost-push inflation." Conversely, an increase in A_1 reduces the production cost, which leads to lower prices and higher output. Finally, an increase in η yields the same result as an increase in A_1 by reducing the markup. These results continue to hold in the case of Cobb–Douglas technology.

B. Neutrality of money

In general, the results of comparative statics depend substantially on whether the model is neoclassical or Keynesian in nature. However, it is not clear which is the Nakatani (2004)'s model. Hence, we examine whether the neutrality of money holds in our model to clarify this point. From (39), (32), and $k = W_1 / W_2$, we can easily derive the equilibrium nominal wages in both sectors (W_1 and W_2), and confirm that they are proportional to the price. That is, the nominal variables (P, W_1, W_2) are proportional to the initial money stock M and the real variables such as output and real wages (W_1 / P and W_2 / P) are independent of M. This gives us the next proposition.

Proposition 5 (neutrality of money)

The neutrality of money holds in our model.

This proposition demonstrates that even if the nominal wage in the primary sector is determined in a non-Walrasian manner, the model is still of the neoclassical type if the nominal wage in the secondary sector, which is determined competitively, is used as the

reservation wage in negotiations. In contrast, as has been pointed out by Dutt and Sen (1997), the model would be Keynesian in nature (that is, the neutrality of money would not hold) if the exogenously determined income such as an unemployment benefit is used as the reservation wage.

C. The properties of welfare of primary and secondary workers

Finally, we examine the properties of welfare of both primary and secondary workers. From (7) and (8), the indirect utility function of each worker is

(40) $\qquad U^{*}_{j} = \tilde{\alpha}\dfrac{W_{j}}{P}.$ $\quad$ ($j = 1,2,$ $\quad \tilde{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$)

This shows that welfare is proportional to the level of the real wage. From (32) and $k = W_{1}/W_{2}$, the real wages in the primary and secondary sectors can be calculated as

$$\frac{W_{1}}{P} = \frac{\eta-1}{\eta}\, aA_{1}\left[a + (1-a)X^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}}, \qquad (\ X \equiv \left(\frac{a}{1-a}\frac{A_{1}}{A_{2}}\right)^{-1}k\ )$$

$$\frac{W_{2}}{P} = \frac{\eta-1}{\eta}\,(1-a)A_{2}\left[aX^{-\frac{\rho}{1-\rho}} + (1-a)\right]^{\frac{1-\rho}{\rho}}.$$

Thus, we obtain the following comparative static results.

$$\frac{\partial U^{*}_{1}}{\partial \beta} > 0, \quad \frac{\partial U^{*}_{1}}{\partial A_{1}} : \text{ambiguous}, \quad \frac{\partial U^{*}_{1}}{\partial \eta} : \text{ambiguous}, \quad \frac{\partial U^{*}_{2}}{\partial \beta} < 0, \quad \frac{\partial U^{*}_{2}}{\partial A_{1}} > 0, \text{ and } \frac{\partial U^{*}_{2}}{\partial \eta} > 0$$

Proposition 6 (properties of welfare of primary and secondary workers)

Under assumption (25), the welfare of primary and secondary workers in equilibrium is given by (40), and it has the following properties:

(a) An increase in the unions' bargaining power β raises (reduces) the welfare of primary (secondary) workers.

(b) An increase in labor productivity in the primary sector A_{1} raises the welfare of secondary workers, but the welfare effect on primary workers is ambiguous.

(c) An increase in the elasticity of substitution between goods η (namely, an increase in the degree of competitiveness in the goods market) raises the welfare of secondary workers, but the welfare effect on primary workers is ambiguous.

Proposition 6 can be explained as follows: An increase in β makes the nominal wage of the primary (secondary) sector relatively higher (lower) through the "relative wage effect" stated in Proposition 1, but it also pulls up the price through an increase in the

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production cost as shown by Proposition 4. Accordingly, the real wage in the secondary sector W_2/P necessarily falls, while the real wage in the primary sector W_1/P increases because the former positive relative wage effect exceeds the latter negative cost effect. The mechanism of the effect of an increase in A_1 is similar. It makes the nominal wage of the primary (secondary) sector relatively lower (higher) through the relative wage effect, while it pulls down the price through a cost reduction. Therefore, W_2/P necessarily increases but the effect on W_1/P is indeterminate because it reduces both the nominal wage and price. Finally, an increase in η makes the nominal wage of the primary (secondary) sector relatively lower (higher) through the relative wage effect, but it also pulls down the price through a lower markup. Thus, W_2/P increases, while the effect on W_1/P is ambiguous.

In the case of Cobb–Douglas technology ($\rho = 0$ or $\sigma = 1$), the real wages in both sectors can be simplified as

$$\frac{W_1}{P} = \tilde{a}\,\frac{\eta-1}{\eta}\,A_1^a A_2^{1-a}\left[1+\frac{\beta}{a(\eta-1)}\right]^{1-a}, \qquad (\tilde{a} \equiv a^a(1-a)^{1-a})$$

$$\frac{W_2}{P} = \tilde{a}\,\frac{\eta-1}{\eta}\,A_1^a A_2^{1-a}\left[1+\frac{\beta}{a(\eta-1)}\right]^{-a}.$$

Via comparative static analysis, we have $\partial U_1^*/\partial A_1 > 0$, which is different from the result (ambiguous) in the case of CES technology detailed above. This is because under Cobb–Douglas technology, an increase in A_1 has no impact on the relative wage, while it pulls down the price by reducing the production cost. Hence, the real wage in the primary sector necessarily becomes higher, which results in an improvement of the welfare of primary workers. The other results in Proposition 6 remain unchanged.

2.4 The Case of Efficient Bargaining

So far, we have discussed the case of the " RTM" model where only the nominal wage in the primary sector is determined through bargaining. In this subsection, we examine the alternative bargaining system called the "efficient bargaining (EB)" where both wage and employment are determined, and show how the results obtained so far change.

The EB model can be formulated as follows:

(41) $\quad \max_{p_i, W_1}$ (20) $\quad$ s.t. (16), (14), and (5)

Note that in the RTM model formulated by (21), the Nash product (20) was maximized with respect to W_{i1} subject to the firm's maximized profit (19), while in the EB model

the Nash product is maximized with respect to both W_{i1} and p_i, subject to the firm's profit function (16) before profit maximization.[7]

Calculating the optimal W_{i1}, we can derive the same first-order condition as (22) in the RTM model, which shows that Proposition 1 and Proposition 2 still hold in the EB model.[8]

In contrast, calculating the optimal p_i, we have

$$(42) \quad p_i (= P) = \frac{\eta}{\eta - (1-\beta)} \frac{W_1}{aA_1} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-\frac{1-\rho}{\rho}}. \quad (X \equiv \left(\frac{a}{1-a} \frac{A_1}{A_2} \right)^{-1} k)$$

Comparing (42) with (32) in the RTM model, the expression of the markup changes from $\eta/(\eta-1)$ to $\eta/[\eta-(1-\beta)]$, which means that in the EB model the markup depends on the unions' negotiation power β. Using (30), (31), and (42), we can derive the labor distribution rates in the EB model as follows:

$$\Omega_1 (\equiv \frac{W_1 L_1}{Pny}) = \frac{a[\eta - (1-\beta)]}{\eta} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1}$$

$$\Omega_2 (\equiv \frac{W_2 L_2}{Pny}) = \frac{(1-a)[\eta - (1-\beta)]}{\eta} X^{\frac{\rho}{1-\rho}} \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{-1}$$

Thus, we can easily show that the sign of $\partial\Omega_1/\partial\beta$ becomes ambiguous in the EB model, whereas in the RTM model its sign is negative (see Proposition 3). The reason for this difference is as follows: In the case of EB, an increase in β produces two contrary effects. First, like in the RTM model, it has a negative impact on the labor distribution rate in the primary sector because the negative relative employment effect exceeds the positive relative wage effect. Second, unlike in the RTM model, it has a positive impact on the labor distribution rate in the primary sector through lowering the markup and the profit distribution rate. These two opposite effects render the sign of $\partial\Omega_1/\partial\beta$ ambiguous. The other results of Proposition 3 remain unchanged.

With respect to output and price in macroeconomic equilibrium, the same results hold between the two bargaining systems; therefore, the results of Proposition 4 and Proposition 5 do not change. In contrast, from (42) and $k = W_1/W_2$, the real wages in

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[7] Note that determining the price is equivalent to determining employment, because the former determines output through (5) and this determines conditional labor demands through (14) and (15).

[8] It is well known that the same first-order condition with respect to the wage holds in both RTM and EB depending on the specification of the functions. See, for example, Nickell (1999).

primary and secondary sectors can be calculated as

$$
\frac{W_1}{P} = \frac{(\eta-1)+\beta}{\eta} \, aA_1 \left[a + (1-a)X^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}},
$$

$$
\frac{W_2}{P} = \frac{(\eta-1)+\beta}{\eta} \, (1-a)A_2 \left[aX^{-\frac{\rho}{1-\rho}} + (1-a) \right]^{\frac{1-\rho}{\rho}}.
$$

Doing comparative static analysis of welfare of primary and secondary workers, we can confirm that the sign of $\partial U_2^* / \partial \beta$ becomes ambiguous in the EB model, which is different from the result ($\partial U_2^* / \partial \beta < 0$) obtained in the RTM model (see Proposition 6). In the EB model, an increase in β produces two contrary effects. First, similar to the RTM model it pulls down the relative wage in the secondary sector. Second, unlike the RTM model it also pulls down the price by reducing the markup. These two opposite effects leave the sign of $\partial U_2^* / \partial \beta$ indeterminate. The other results in Proposition 6 continue to hold.

Summarizing so far, we have the following:

Proposition 7 (different bargaining systems)

By changing the bargaining system from RTM to EB, the effect of an increase in the unions' negotiation power β on the labor distribution rate in the primary sector Ω_1 changes from positive to ambiguous, and the effect of it on the welfare of secondary workers U_2^* changes from negative to ambiguous. The other results do not change.

3. Comparison with Nakatani (2004)

As mentioned in the introduction, the main purpose of our study is to re-examine the results of Nakatani (2004) by using a more rigorous general equilibrium framework. Hence, we should explain in detail the differences between our results and his.

We summarize our comparative static results of the RTM model in Table1.

Table 1: Comparative static results of our paper

	W_1/W_2	L_1/L_2	Ω_1	Ω_2	y
β	+	−	−	+	−
A_1	−	+	+	−	+
η	−	+	+	?	+

The results of Nakatani (2004) are summarized in Table 2.[9],[10] Here, note that the signs in parentheses mean that they are ambiguous in general and are therefore determined by numerical analysis (namely, by substituting some plausible values into exogenous parameters).

Table 2: Comparative static results in Nakatani (2004)

	W_1 / W_2	L_1 / L_2	$w_1 L_1$	$w_2 L_2$	y
β	+	−	−	+	0
A_1	(−)	+	+	−	+
η	(−) for small β	(−) for large β	+	(+)	+

Comparing the two tables, we can see that our results are similar to his, but there are some important differences detailed below:

(A) We derive clear comparative static results on $\partial(W_1 / W_2)/\partial A_1$, $\partial(W_1 / W_2)/\partial \eta$, and $\partial(L_1 / L_2)/\partial \eta$ without depending on the numerical analysis used by Nakatani (2004), which implies that our results are more robust than his.

(B) On the effect of an increase in the unions' negotiation power β on output y, Nakatani (2004) asserted that it had no impact because of the assumption that the propensities to consume of both primary and secondary workers are the same. However, we prove that it has a positive impact even if they are the same because an increase in β pulls up the price and lowers output through the negative real balance effect.

(C) Nakatani (2004) calculated $\partial(W_1 / W_2)/\partial \eta$ and $\partial(L_1 / L_2)/\partial \eta$ numerically under different values of β and then concluded that both signs were negative. We show, however, that the former (latter) is negative (positive), independent of the values of β. In our model, they always have a negative correlation because of the firm's cost minimization behavior.

(D) Nakatani (2004) considered only the case of RTM, while we investigate both RTM and EB. In the latter case, some results of Proposition 3 and 6 changed, because the firm's markup becomes dependent on β.

(E) It is not clear whether the Nakatani (2004) model is neoclassical or Keynesian in nature. In our study, in contrast, by adopting a monetary economy model, we check whether the neutrality of money holds and then show that our model is neoclassical in

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[9] Nakatani (2004) studied only the case of RTM.

[10] Note that Nakatani (2004) examined the effects on the *levels* ($w_1 L_1$ and $w_2 L_2$) of labor distribution, not the *rates* (Ω_1 and Ω_2).

nature. Furthermore, we examine the effects of exogenous shocks on price and individual welfare, which are aspects that are not discussed in Nakatani (2004).

4. Concluding Remarks

Using a static general equilibrium model with monopolistic competition and labor unions, we re-examined in detail the results of Nakatani (2004), who studied the characteristics of a dual labor market economy in a rather simple macroeconomic model.
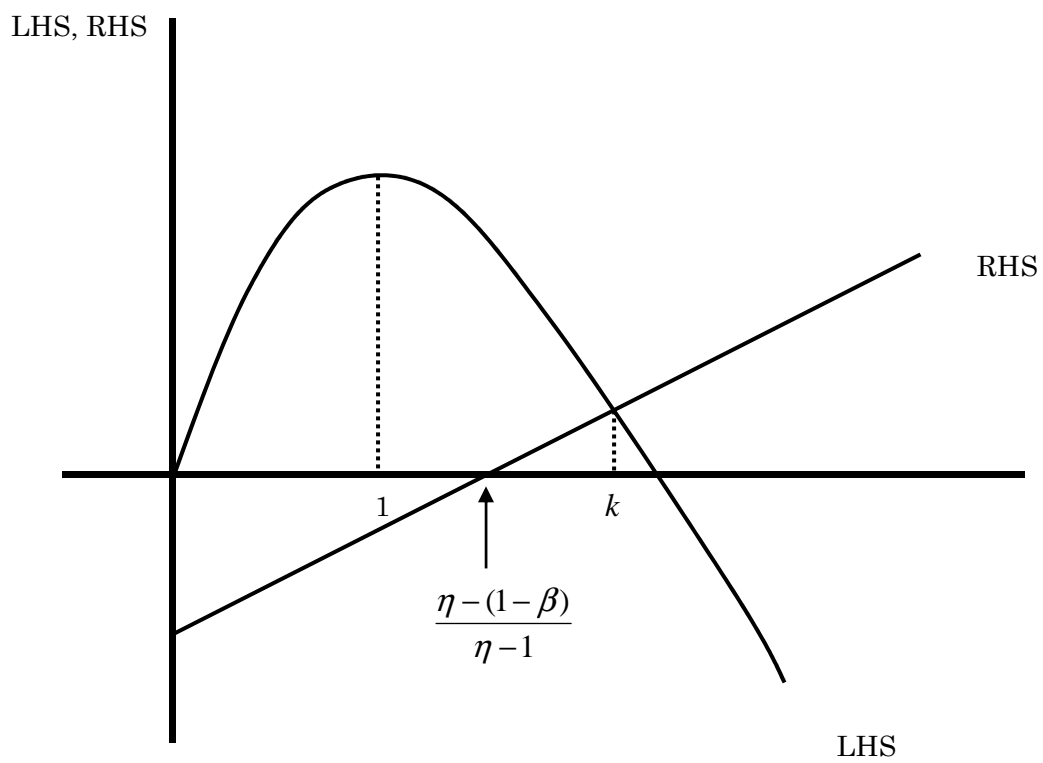
The differences between our results and Nakatani's are summarized in the previous section. Our results are similar to his, but there are some important differences. We also examined aspects that were not studied by him.

Our study examines the characteristics of a dual labor market under the simplified setting that workers who are not employed in the primary sector move to the secondary sector without hesitation. In reality, however, these two sectors are often clearly segmented. Those who lose their jobs in the primary sector do not move to the secondary sector and instead remain unemployed, while potential workers in the secondary sector, for example, married women, often voluntarily stay in the secondary sector in consideration of their work–life balance. Yoshikawa (1995) presents a simple dual labor market model considering this aspect of a dual labor market. Introducing this factor in our model is one of the interesting questions.

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(Figure 1: equation (22))